Appendix: Method for alignment of multiple shapes of the same residuum

We created a reference limb surface for each subject's residuum by averaging surfaces from all available scans and then finding the surface closest to the average in a least-squares Euclidean distance sense. To achieve this, we first aligned all available limb surfaces for the subject using the patellar tendon bar as a reference mark. The average shape was then determined. Then we determined the surface that produced the minimal root-mean-square error between the spline fit for the surface and the average shape. This surface was defined as the "reference limb surface." Our basis for using this method was so that the reference limb surface was a representative average of all scans, as opposed to being at an extreme. Using this technique, we reduced the errors in calculating shape differences.

Each individual limb surface was then aligned with respect to the reference limb surface. For each individual surface, the alignment was performed in two stages. First, the mean (unsigned) distance between corresponding points on the surfaces was minimized. For a dense, uniformly distributed set of points, this procedure is analogous to minimizing the volume of the symmetric difference between the two surfaces. In the second stage, we further refined the initial alignment of volumes to localize any shape differences at the surfaces. In this stage, we maximized the shape similarity, at the expense of volume difference if needed. We did this by conceptually dividing the surfaces into small quadrangular facets and then minimizing the differences between the local tangents of corresponding facets in each surface pair, thereby allowing regions of like shapes to align with each other. In the two-dimensional case, this concept is analogous to dividing two curves into short segments and then comparing the tangents of each segment pair, with the goal of minimizing the net difference. For the present three-dimensional case, we implemented this concept by maximizing the dot-product of corresponding surface normals; the dot-product of two parallel vectors is 1.0 and that between two perpendicular vectors is 0.0. Since the surfaces started out very similar, all dot-products were already very close to 1.0. Therefore, to amplify the differences in shape, we instead maximized the mean arc hyperbolic tangent of the dot-products.

After we aligned all limb surfaces, we could then calculate limb volume and shape differences between any pair of surfaces.