Computational method to evaluate ankle postural stiffness with ground reaction forces

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Abstract—We examined an existing method for evaluating postural sway based on force-plate technology. Through an improved mathematical model of postural dynamics, we propose a new method, which better evaluated postural sway and, in addition, computed ankle moment and ankle postural stiffness directly from the measured ground reaction forces. An example is detailed that demonstrates the utility of this approach. The proposed method does not involve filtering or numerical integration and considers the platform inclination. Results from normal subjects show a linear relation between the ankle moment and the sway angle during quiet standing.

Key words: ankle postural stiffness, postural stability, quiet standing.

INTRODUCTION

Postural control is the control needed to maintain the posture during upright standing. This control is coordinated by the central nervous system with input from three systems: visual, vestibular, and somatosensory (or the proprioceptive system). Body sway is used to indicate postural stability. Various neurological and musculoskeletal diseases are related to impaired balance, which results in an increased risk of falling caused by deficits of the proprioceptive system or muscle weakness. Falls caused by impaired postural control present a serious health hazard to the elderly as well as to persons with balance disorders. Impaired balance diminishes a person’s ability to perform activities of daily living.

Three test protocols are used in clinical Computerized Dynamic Posturography developed by NeuroCom for diagnosing the functional impairments underlying balance disorders: The first protocol is the Sensory Organization Test. It is intended to assess the patient’s ability to effectively use visual, vestibular, and somatosensory information and to appropriately suppress disruptive visual and/or somatosensory information under sensory conflict conditions. The Motor Control Test, the second protocol, is intended to assess the patient’s ability to reflexively recover from unexpected external provocations quickly and with appropriate movement patterns. The Adaptation Test, final protocol, is intended to assess the ability to modify reflexive motor reactions when the support surface is irregular or unstable. Most other Computerized Dynamic Posturography devices quantify postural stability using force-plate technology. These devices measure the ground reaction forces with transducers attached to a force plate to determine the center of pressure (COP).

Abbreviations: COM = center of mass, COP = center of pressure, SD = standard deviation.

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The upward projection of the COP is used as an estimate for the body center of mass (COM). Different low-pass filters are used on the COP time series to remove the high-frequency content [1,2], based on the assumption that postural sway is quasi-static. Another approach is to estimate COM with the second integral of horizontal acceleration, which is assumed to be proportional to the horizontal ground reaction force [3]. However, this method requires the estimation of initial conditions [4]. Winter et al. estimate COM based on their 14-segment COM model and measurements at 21 markers [5]. While this approach is good for research, it is less practical for clinical use.

Another reason for obtaining COM is for evaluating the ankle postural stiffness [5,6]. This evaluation requires determining moment produced at the ankle for maintaining posture. In most of the studies, only the moment equilibrium was considered, whereas equilibrium in horizontal and vertical directions is ignored in the system equations.

This study was aimed at evaluating ankle postural stiffness based on balance test data collected with a NeuroCom’s EquiTest device [7]. (Disclaimer: Not one of the authors has a vested interest in the EquiTest or NeuroCom.) This system was developed to run on the early personal computers of the 1980s, and consequently, its calculations are as numerically simple as possible. The operating principle of this device is described in Appendix A, which can be found in the on-line version only. During our investigation, we noticed several shortcomings in the device-generating COM. One shortcoming was that this device uses the moving average of the COP as an estimate for COM. As mentioned earlier, this estimation is only good for quasi-static standing, while some of the test conditions should be treated as dynamic, because most individuals show considerable sway, particularly when the platform moves. Another problem is that the shear force, although measured, is not used to estimate COM. Thus, the computation performed by the device does not consider rotation of the force plate, which as we will show, produced an incorrect estimation of the COM.

Our work intended to correct the shortcomings just mentioned by developing a mathematical model for quantifying COM directly from the measurement of ground reaction forces, while considering the rotation of the force platform. We also propose using the model to study the stiffness of the muscles around the ankle and their relation to the destabilizing force of gravity on the human body. The use of our computational method to generate COM, ankle stiffness, and other information is demonstrated with an example.

**METHOD**

As described in Appendix A (on-line version only), an EquiTest device generates balance test results in two steps. First, its data acquisition hardware produces ground reaction forces from five transducers, sampled at 100 Hz. Then, its software processes the time series of force measurement to generate various reports for each trial of every test condition (see Appendix A [on-line version]). The reported manufacturer’s specification for the resolution of a force transducer, together with its amplifier and analog-to-digital conversion, is 0.87 N (0.195 lb). However, we believe the calculations made by the device have shortcomings. Thus, our approach was to develop a new computational method based on the output of the data acquisition hardware (i.e., the quantization levels of the force transducers). We then used the results to evaluate COM, ankle moment, ankle muscle stiffness, and net moment around the ankle.

To better understand the calculations made by the device and the problem therein, we first reviewed the dynamics of the “human inverted pendulum” for sway in the sagittal plane. Figure 1 shows the entire body excluding feet as an inverted pendulum rotating about the ankle joint A. $M$ is the mass of body above the ankle, $F_{H,A}$ and $F_V$ are horizontal and vertical forces acting at the ankle joint, $\tau$ is the resultant moment acting at the ankle joint by muscles and passive structures (ligaments, cartilage) around the ankle, $g$ is the gravitational constant, and $\theta$ is absolute sway angle with respect to a fixed vertical reference.

Figure 2 shows the feet together with the force plate. $m$ is the total mass of the feet and the force plate; $F_F$ and $F_R$ are ground reaction forces perpendicular to the force plate, measured with front and rear transducers, respectively; $F_H$ is the ground reaction force parallel to the force plate, measured with a transducer at the pin joint; $d$ is the distance between the pin axis and the transducers that measure forces perpendicular to the force plate; $e$ is the distance between the ankle joint and the top surface of the force plate; $a$ (not shown) is the perpendicular distance between the line through the ankle and pin joints and the COM of the feet; $\theta_m$ is the sway angle of the COM relative to the line perpendicular to the force plate; and $\phi$ is the inclination angle of the force plate.
The calculation by the device does not consider the inclination, \( \phi \), of the force platform and the shear force, \( F_H \). In fact, the only use of shear force measurement by the device is to obtain the “strategy score,” which is viewed as an indicator of the involvement of hip sway (instead of ankle sway) in maintaining balance. We believe the contribution by the inclination of the platform to shear force should not be ignored. Let us illustrate the effect of inclination on shear force with the sample data shown in Appendix A (on-line version). For data point 1998, we have \( F_F + F_R = 196 + 256 + 246 + 151 = 849 \) (quantization levels) and \( F_H = 86 \) (quantization levels). The ratio \( F_H / (F_F + F_R) = 0.080 \) is the result of sway dynamics and the inclination of the force platform. The contribution from the inclination of the force platform may be seen from static situations. If the inclination of the force platform is \( \phi \), then \( F_H / (F_F + F_R) = \tan \phi \) in static situations. It may be noted that if \( \phi = 2^\circ \), \( \tan \phi = 0.035 \), and if \( \phi = 5^\circ \), \( \tan \phi = 0.087 \). Comparing these values to the ratio of 0.080 in the above example, we see that the inclination of the force platform could contribute a significant portion of the total shear force.

To address the problem of rotation not considered in the computation, we first derived a complete set of dynamic equilibrium equations (1) to (3) to establish the relationship between sway movement and the ground reaction forces:

\[
M_h(\dot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta) = F_H \cos \phi + (F_F + F_R) \sin \phi \quad (1)
\]

\[
M_h(\dot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta) = (M + m)g - (F_F + F_R) \cos \phi + F_H \sin \phi \quad (2)
\]

\[
I \ddot{\theta} = Mgh \sin \theta - (F_F - F_R) d - F_H e + mga \cos \phi \quad . \quad (3)
\]

Parameters \( M, m, I \) (the body’s moment of inertia about ankle joint), \( h \) (the distance between COM and the angle joint), \( e \), and \( a \) in the equations are functions of a subject’s height and weight and are calculated with the use of anthropometric data taken from the literature [8–10]. Equations (1) to (3) can be found in Appendix B [found in the on-line version only] as equations (B10), (B11), and (B12).

The set of dynamic equations was then solved to obtain equation (4)

\[
\begin{align*}
&\left[ (M + m) - M^2 h^2 \right] g \sin \theta - (F_F + F_R) \sin \frac{\theta}{k+1} + F_H I \cos \frac{\theta}{k+1} \\
&- M_h m a \cos \frac{k\theta}{k+1} + M_h ((F_F - F_R) d + F_H e) = 0 \quad . \quad (4)
\end{align*}
\]
which is a nonlinear equation involving the absolute sway angle, $\theta$. A simple solution was obtained through the application of a small angle approximation in equation (4).

$$\theta = \frac{Mh[(F_F - F_R)d + F_{HE}e - mga] + I \cdot F_{HE}}{M^2gh^2 - I(2(M + m)g - \frac{F_F + F_R}{k + 1})}.$$  \hspace{1cm} (5)

We have compared the solution obtained with and without the approximation, and the approximation resulted in negligible change. (You can find equations (4) and (5) as equations (B15) and (B19), respectively, in Appendix B on-line.) The error generated by the small angle approximation was less than 0.13, 0.51, and 1.2 percent for $\theta$ of $5^\circ$, $10^\circ$, and $15^\circ$, respectively. The relative sway angle, if desired, can be obtained as $\theta_m = \theta/k + 1$ (gain $k$ is the ratio of the rotation angle of the base to the sway angle $\theta$).

The EquiTest device we used did not produce information on the ankle moment. In our computation, we included the evaluation of the ankle moment so that we could study the relationship between the moment and angular sway at the ankle joint, which is shown as

$$\tau = (F_F - F_R)d + F_{HE}e - mga \cos \frac{k\theta}{k + 1}.$$  \hspace{1cm} (6)

We used only the portion of the ankle moment responsible for the elastic deformation at the ankle joint to determine ankle postural stiffness. (Equation (6) is equation (B20) in Appendix B on-line.) Our results suggest that a linear relation may exist between the ankle moment and the sway angle when the platform is fixed. This led us to perform a linear regression of the ankle moment versus the sway angle, in the form of $\tau = k_p\theta + k_d\theta + k_c$, to the test data obtained under such conditions with the platform fixed. The term $k_p$ is the elastic component of the ankle moment, $k_d\theta$ is the viscous component of the ankle moment, and $k_c$ represents the constant component. Results of this regression are discussed in the following sections. Since our results show little correlation between $\tau$ and $\theta$, the relation $\tau = k_p\theta + k_d\theta + k_c$ reduces to $\tau = k_p\theta + k_c$. This correlation implies that the viscous component in the ankle moment is negligible. The slope $k_p$ of the linear regression is thus the ankle postural stiffness during quiet standing.

Postural sway is the result of interaction between the ankle moment and the destabilizing moment $\tau_g$ of gravity. The time history of the net moment might be able to reveal additional information about the sway. For this reason, we also looked into the time history of the net moment $\tau_g = \tau$.

We implemented our new computational method with MATLAB. The input to our MATLAB program is the data files, which contain the quantization levels of force transducers, generated by an EquiTest device. We tested this program with data from four healthy adult subjects (two males and two females, ages between 29 and 70). One of the subjects is an author of this paper. Other subjects provided informed consent using forms approved by the University of Medicine and Dentistry of New Jersey Institutional Review Board. Each of the four sets of complete Sensory Organization Test data contains eighteen 20 s trials (three trials for each of the six conditions, see Appendix A [on-line version]).

When the computational method is used to process clinical data, a detailed error analysis will be needed to determine the degree of precision in the computed sway angle and the ankle moment caused by the errors in force measurement. We did not perform error analysis in this study, because the numerical results presented in the following section are used only to illustrate the use of our method.

RESULTS

This study focuses on the method to be used for the evaluation of a subject, not on the specific results from any individuals or groups. Since similar results were obtained for all four subjects, only the results from one subject (one of the authors) are presented here to illustrate the use of our method. This male subject’s height and weight are $H = 1.67$ m and $W = 740.66$ N, respectively. Other parameters for the subject are found as $M = 73.235$ kg, $m = 2.265$ kg, $I = 85.02$ kg · m$^2$, $h = 0.933$ m, $d = 0.107$ m, $e = 0.065$ m, and $a = 0.0315$ m. Two of the eighteen trials are presented here. One represents the test conditions where the platform is fixed ($k = 0$), and the other represents the test conditions where the platform is “sway referenced” and rotates the same angular amount that the COM moves ($k = 1$).

Figure 3 shows our computed COM ($y = h \cdot \theta$), labeled as “our result,” and the device-reported COM, labeled as “moving average,” for one trial with $k = 0$. Although both curves are based on the same measured forces, the computed curve differs quantitatively from the curve generated by the device because the computational methods are different. Since the force platform has no inclination and the shear force is negligible when the platform is fixed, the difference between the two curves
is relatively small. The device estimates $h$ as $0.5527H$ or $h = 0.923m$, which is slightly smaller than the value ($h = 0.933m$) we used. As a result, the device-reported COM curve is slightly lower (about 1%) than our computed COM. We can see clearly the smoothing effect of the moving average in the device-reported COM.

Figure 4 shows our computed COM, both the absolute sway $y = h \cdot \theta$ (with respect to a fixed vertical reference) and the relative sway $y_m = h \cdot \theta_m$ (relative to the line perpendicular to the force plate), and the device-reported COM for one trial with $k = 1$. The COM reported by the device is also relative to the line perpendicular to the force plate. Because of the rotation of the platform, this trial involves the inclination of the force platform and a noticeable shear force. The difference between our computed COM and the device-reported COM can clearly be seen and is due to the inclusion of the shear force and the rotation of the platform in our analysis.

The time series of the computed ankle moment for the same two trials (platform fixed and moving) is plotted separately in Figures 5 and 6. When we compare the plots of COM in Figure 3 and ankle moment in Figure 5, the similarity of the two plots is apparent. This similarity is consistent among all other subjects when the platform is fixed. When we compare the plots of COM in Figure 4 and ankle moment in Figure 6, the plot of ankle moment is similar to the plot of the device-reported COM, but it has no similarity to the computed COM.

The result of the linear regression of the ankle moment versus the sway angle for the trial presented in Figure 3 is shown in Figure 7. The correlation between $\tau$ and $\theta$ is confirmed by coefficient $r_{\tau \theta} = 0.996$, but there is little correlation between $\tau$ and $\dot{\theta}$, since coefficient $r_{\tau \dot{\theta}} = -0.059$. The equation of the line is $\tau = 655.73 \cdot \theta + 0.978$ (N · m) with the coefficient of determination $R^2 = 0.993$. The ankle postural stiffness is therefore 655.73 N · m/rad.
for sway with the fixed platform. The goodness-of-fit of the linear regression can be estimated with the chi-square probability of the fit, which requires the knowledge of the measurement errors and their distribution [11]. A simple means we used to evaluate error in parameters (the slope and the intercept) obtained from linear regression is the standard deviation (SD) of the fit, which is approximately the average difference between each data point and the best-fit line. In the case of Figure 7, the SD of the fit is 0.10 N·m. With the ankle moment between 40.4 and 46.1 N·m during the trial, the SD of the fit is less than 0.25 percent. We can also check the SD of the slope and the SD of the intercept, which are approximately the difference in slope and intercept, respectively, between the best-fit line and a limiting reasonably fit line. The SD for the slope is 1.23 N·m/rad, which is about 0.2 percent of the slope. The SD for the intercept is 0.08 N·m, which is about 8 percent of the intercept.

The moment produced by the gravitational force can be represented by another line \( \tau_g = Mgh \cdot \theta = 670.22 \cdot \theta \) (N·m). The time history of the net moment \( \tau_g - \tau \) for the same trial is presented in Figure 8. It shows that the net moment is quite close to zero during the entire trial. However, the bit error in the force measurement makes the actual values of the net moment meaningless. The discrete nature of the force measurement because of analog-to-digital conversion was masked in the plotted ankle moment but is revealed in this plot. If the true nature of the net moment is to be revealed, a much-improved resolution in force measurement is needed.

The Table shows one example of regression of the ankle moment versus the sway angle for each of the four subjects, all under the condition that the platform is fixed and eyes open. All the regression results show high values of the coefficient of determination. We can conclude from these results that the ankle moment varies linearly with sway angle for all four subjects when the platform is fixed.

![Figure 6](image1.png)

**Figure 6.**
Computed ankle moment for trial shown in Figure 4.

![Figure 7](image2.png)

**Figure 7.**
Ankle moment versus sway angle for trial shown in Figure 3.

![Figure 8](image3.png)

**Figure 8.**
Net moment \( Mgh \cdot \theta - \tau \) at ankle for trial shown in Figure 3.
DISCUSSION

The computational method we developed has two important features. One is that the solution was obtained without either filtering or numerical integration. The other is the inclusion of shear force and rotation of the platform. This computational method can also be applied to situations where the platform is fixed and inclined (i.e., \( \phi \) is a nonzero constant). In our computations, we do not ignore parameters \( F_H, e, m, \) and \( a \) as was done in the previous studies by others, such as Winter et al. and Morasso and Sanguineti [5,12].

Several factors can affect the results obtained with our computational method. One factor is the accuracy of ground reaction force measurement. For the NeuroCom device we used, the resolution of the force measurement is about 0.87 N for each transducer. This limited resolution leads to significant error when the value of quantization levels is small. For example, the back-and-forth sway motion indicates the existence of angular acceleration. However, meaningful calculation of angular acceleration cannot be achieved with the current resolution of force transducers when the platform is fixed. Because the sway angle is small, the magnitudes of the two terms \( (\phi - (F_F + F_R)\sin \theta) \) and \( F_H \cos \theta \) in equation (B13) (with \( \phi = 0 \)) (Appendix B on-line) are really in ranges comparable to each other. When the platform is fixed, \( F_H = 0 \) is reported and \( B = 0 \) in equations (B15) to (B17) (Appendix B). The values of \( \sigma = \pm 1 \) in equation (B18) produce two supplementary solutions of angle \( \theta \). Another factor is the potential rotation at other joints. Our computation is based on the simple inverted pendulum model of postural sway, which assumes that the rotation happens only at ankle joints. While we observed no obvious hip movement during our experiments, this assumption requires further validation through experiments with multilink models. This validation, which is the subject of our future research, will help to better define the operating range of the device.

The moving average displacement in Figure 4 is closer in shape to the ankle moment in Figure 6 than the displacement computed with our method. We might expect this result if we approximate the moving average displacement by ignoring the horizontal force component in equation (B19) and compare it with the ankle moment in equation (B20) (Appendix B). The need to account for shear forces in the computation becomes evident when we look closely at equations (B19) and (B20). Let us again use the data point 1998, from one trial of our example subject, in Appendix A (on-line version). We have \( F_F - F_R = 196 + 151 - 256 - 246 = -155 \) (quantization levels) and \( F_H = 68 \) (quantization levels). The two equations produce \( \theta = 0.302 \) (rad) = 17.3° and \( \tau = -11.3 \) (N · m). If \( F_H \) is ignored, we obtain \( \theta = -0.069 \) (rad) = -4.0° and \( \tau = -15.1 \) (N · m) instead. Thus ignoring \( F_H \) leads to a significant difference in the results. The different shapes of the ankle moment and the displacement computed with our method suggest that a linear relationship between them no longer holds when the plate is rotating. How the moment of the shear force and the inertia moment of the body interact at the ankle joints requires further investigation, if the simple inverted pendulum model is not applicable. Joint position data for multiple joints obtained from a system other than the balance test will be needed to develop such a model.

We conclude from Figures 5 and 6 that very different ankle moments are required, depending on whether the platform is fixed or moving. The maximum ankle moment generated with the moving platform is almost double that generated with the fixed platform in these two trials. It is also observed that negative ankle moment is generated with the moving platform in contrast to the fixed platform where there is no negative moment.

Test conditions with the platform fixed represent the quiet standing studied by Winter et al. [5,6]. Our results for ankle moment show that the ankle stiffness closely resembles an ideal spring, which is in agreement with Winter et al. [6]. In our linear regression, both ankle moment and sway angle are calculated from the same measured ground reaction forces. Values of the ankle stiffness and the coefficient of determination \( R^2 \) vary in different trials and conditions. The coefficient of determination reported by Winter et al. is \( R^2 = 0.954 \) [6], where ankle moment and sway angle were obtained from separate measurements. Both the “moving average” and “our result” would produce similar fit from the linear regression. What is gained with our method is that the details of the sway and the ankle moment are not being smoothed out. These details could become useful in finding better outcome measures of postural balance.

### Table

Sample regression data of four subjects.

<table>
<thead>
<tr>
<th>Subject</th>
<th>( Mgh ) (N · m/rad)</th>
<th>( \tau ) (Regression) (N · m)</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>644.122</td>
<td>640.325·( \theta ) + 0.195</td>
<td>0.989</td>
</tr>
<tr>
<td>2</td>
<td>600.927</td>
<td>596.323·( \theta ) + 0.294</td>
<td>0.995</td>
</tr>
<tr>
<td>3</td>
<td>917.306</td>
<td>912.361·( \theta ) + 0.231</td>
<td>0.999</td>
</tr>
<tr>
<td>4</td>
<td>670.216</td>
<td>670.216·( \theta ) + 0.978</td>
<td>0.993</td>
</tr>
</tbody>
</table>
Although the slope 655.73 N·m/rad of the regression line in Figure 7 differs from that (670.22 N·m/rad) of the moment $\tau_g$ produced by gravity, the net moment $\tau_g - \tau$ is less than 1 percent of the ankle moment, as shown in Figure 8. Intersection of these two moment lines occurs at $\theta = 0.0675$ (rad) = 3.87° for the trial in Figure 3. The corresponding COM displacement is 0.063 m, which represents a critical point of the stability. Thus the sway motion should not be allowed to deviate too much beyond this point to ensure stability.

Our computation of the COM for sway-referenced motion (platform rotating) produces significantly different results when compared with those reported by the device. We believe the computation by the machine is incorrect for these conditions, since it ignores the effect of the shear force as well as the mass and the rotation of the force plate. As long as the simple inverted pendulum model is still appropriate for the sway-referenced motion, our method will produce the correct results. In our formulation, we assume that the rotation of the force plate is precisely servo-controlled to follow the sway of the subject. In reality, there should be a time delay of one sampling period (10 ms in this case) in obtaining the sway angle for position control. For sway-referenced motion (platform rotating), we simply showed the result of the calculation under the assumption that the model is applicable. The relationship between the ankle moment and the sway angle during sway-referenced motion requires further study.

CONCLUSION

The new computational method corrects shortcomings in an existing method for evaluating postural stability by including inclination of the platform and the shear component of the ground reaction force in the mathematical model. Based on this model, the solution of postural sway is obtained without either filtering or numerical integration. In this method, ankle moment and ankle postural stiffness for quiet standing are also evaluated. Factors that could affect the application of this method are discussed.

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