Appendix B: Derivation of the Computation Method

In this appendix, we present a derivation of the formulas for the proposed computational method. Nomenclature used here is introduced in connection with **Figures 1** and **2** of the main paper.

The equation of motion for the body in **Figure 1** of the main paper can be written as

$$F_{H,A} = M\ \ddot{x}_c = M\ h(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta) \tag{B1}$$

$$F_V = M(g + \ddot{y}_c) = Mg - Mh(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta) \tag{B2}$$

$$-\tau + F_V h\sin\theta - F_{H,A} h\cos\theta = I_c\ddot{\theta}\,, \tag{B3}$$

where $\ddot{x}_c$ and $\ddot{y}_c$ are respectively the horizontal and vertical acceleration at COM, I_c is the moment of inertia about the COM, g is the gravitational constant, τ = resultant moment acting at ankle joint by muscles and passive structures (ligaments, cartilage) around ankle, and h is distance between COM and the ankle joint. By substituting equations (B1) and (B2) into (B3) and simplifying, we have

$$-\tau + Mgh\sin\theta = (I_c + Mh^2)\ddot{\theta} = I\ddot{\theta}\ \ , \tag{B4}$$

where I is the moment of inertia about ankle joint.

The equation of motion for the feet in **Figure 2** of the main paper can be written as

$$F_{H,A} = F_H\cos\phi + (F_F + F_R)\sin\phi \tag{B5}$$

$$F_V = (F_F + F_R)\cos\phi - F_H\sin\phi - mg \tag{B6}$$

$$\tau - (F_F - F_R)d - F_H e + mga\cos\phi = 0\ . \tag{B7}$$

The sway angle computed by the device is θ_m, which is based on the reading of the vertical force transducers, e is the distance between the ankle joint and the top surface of the force plate, a is the perpendicular distance between the line through the ankle and pin joints and the COM of the feet. Its reference is the line perpendicular to force plate. As mentioned in **Appendix A**, the inclination angle ϕ is related to the computed sway angle θ_m with a gain factor (–1.0 ~ 2.0) as

$$\phi = k\cdot\theta_m\ . \tag{B8}$$

The absolute sway angle θ can be expressed as

$$\theta = \phi + \theta_m \,. \tag{B9}$$

After eliminating all the internal forces and moments $F_{H,A}$, F_V and τ from equations (B1–B7), we have the following three equations for the postural system:

$$Mh(\ddot{\theta}\cos\theta - \dot{\theta}^2 \sin\theta) = F_H \cos\phi + (F_F + F_R)\sin\phi \tag{B10}$$

$$Mh(\ddot{\theta}\sin\theta + \dot{\theta}^2 \cos\theta) = (M+m)g - (F_F + F_R)\cos\phi + F_H \sin\phi \tag{B11}$$

$$I\ddot{\theta} = Mgh\sin\theta - (F_F - F_R)d - F_H e + mga\cos\phi \quad . \tag{B12}$$

Since the ground reaction forces F_F, F_R and F_H are measured with the force transducers and the base rotation ϕ is under specified machine control, we can obtain the unknown state θ, $\dot{\theta}$, and $\ddot{\theta}$ of the sway at each sampling point by solving equations (B10–B12). From equations (B10) and (B11), we have

$$\ddot{\theta} = \frac{(M+m)g\sin\theta + (F_F + F_R)\sin(\phi-\theta) + F_H \cos(\phi-\theta)}{Mh} \tag{B13}$$

$$\dot{\theta}^2 = \frac{(M+m)g\cos\theta - (F_F + F_R)\cos(\phi-\theta) + F_H \sin(\phi-\theta)}{Mh} \,. \tag{B14}$$

Note that the solution only yields the magnitude $\left|\dot{\theta}\right|$, not the direction, of the angular velocity $\dot{\theta}$. Therefore, this direction information must be derived from the time history of angle θ. Substituting (B13) into (B12), and using $\phi = \frac{k}{k+1}\theta$, we have

$$\begin{aligned}&\left[I(M+m) - M^2h^2\right]g\sin\theta - I(F_F + F_R)\sin\frac{\theta}{k+1} + F_H I\cos\frac{\theta}{k+1} \\ &\qquad - Mhmga\cos\frac{k\theta}{k+1} + Mh\left[(F_F - F_R)d + F_H e\right] = 0\end{aligned} \quad . \tag{B15}$$

For the test conditions where the platform is fixed, we have $k = 0$ and equation (B15) becomes

$$\left[I(M+m)g - I(F_F + F_R) - M^2gh^2\right]\sin\theta + F_H I\cos\theta + Mh\left[(F_F - F_R)d + F_H e - mga\right] = 0 \,, \tag{B16}$$

which can be expressed as

$$A\sin\theta + B\cos\theta + C = 0 \;, \tag{B17}$$

where

$$A = I \cdot \left[(M+m)g - (F_F + F_R)\right] - M^2 h^2 g \ , \ B = F_H I \ ,$$

and

$$C = Mh\left[(F_F - F_R)d + F_H e - mga\right].$$

The solution to equation (B17) can be found by making substitutions $t = \tan\frac{\theta}{2}$, $\sin\theta = \frac{2t}{1+t^2}$, and

$\cos\theta = \frac{1-t^2}{1+t^2}$, as

$$t = \tan\left(\frac{\theta}{2}\right) = \frac{-A + \sigma\sqrt{A^2 + B^2 - C^2}}{C - B} \ , \qquad \text{(B18)}$$

where $\sigma = \pm 1$ identifying the solution mode. The location of the COM can be obtained as

$$y = h\sin\theta = h\frac{2t}{1+t^2} \ .$$

For the test conditions where the platform is rotating, we have $k \neq 0$ and the solution to equation (B15) involves more complex computation. A small angle approximation may be employed to simplify the computation. With this approximation, equation (B15), reduces to

$$\theta = \frac{Mh[(F_F - F_R)d + F_H e - mga] + I \cdot F_H}{M^2 g h^2 - I\left[(M+m)g - \frac{F_F + F_R}{k+1}\right]} \ . \qquad \text{(B19)}$$

The ankle moment can be evaluated with equation (B7) for all test conditions as

$$\tau = (F_F - F_R)d + F_H e - mga\cos\frac{k\theta}{k+1} \ . \qquad \text{(B20)}$$