APPENDIX: Moment of Inertia Calculations

The following experiment and calculations were used to determine the moment of inertia of the right roller of the dynamometer and the five different tires (each mounted on a 0.61 m Sunrims SW600 wheel) used in this study. For convenience, only the testing of the roller is described; a similar setup and test protocol were used to determine the moment of inertia of the empty wheel (without a tire). A nylon string was secured around the circumference of the roller and a series of weights (ranging from .048 kg to .921 kg) was attached to the end of the string. The initial energy of the system (E_i) can be described by

$$E_i = mgh_i + \frac{1}{2}mv_i^{\,2} + \frac{1}{2}I\omega_i^{\,2}, \qquad \textbf{[A1]}$$

where

m	=	The mass of the weight.
g	=	The gravitational acceleration.
h_i	=	The initial height of the weight.
v_i	=	The initial velocity of the weight.
I	=	The moment of inertia of the roller.
ω_i	=	The initial angular velocity of the roller.

Assuming mechanical energy is conserved (minimal energy loss in the bearings and string), the final energy of the system (E_f) is equal to the initial energy of the system:

$$E_f = mgh_f + \frac{1}{2}mv_f^{\,2} + \frac{1}{2}Iw_f^{\,2} = mgh_i + \frac{1}{2}mv_i^{\,2} + \frac{1}{2}Iw_i^{\,2} \quad , \qquad \textbf{[A2]}$$

where the subscript *f* denotes the final values of height, velocity, and angular velocity.

The angular velocity of the roller is related to the linear velocity of the weight by

$$\omega = \frac{v}{R} \quad , \qquad \textbf{[A3]}$$

where R is the radius of the roller. By substituting **Equation A3** into **Equation A2**, the moment of inertia of the roller can be computed by

$$I = mR^2 \left[\frac{2g(h_i - h_f)}{(v_f^{\,2} - v_i^{\,2})} - 1 \right]. \qquad \textbf{[A4]}$$

During each trial, the position of the weight was tracked with a reflective marker, placed on top of the weight, and a three-camera Vicon system. The vertical velocity of the weight was determined using the central difference calculation:

$$v = \frac{h(t+1) - h(t-1)}{2\Delta t} \quad . \textbf{[A5]}$$

To smooth the point to point differences in velocity and provide a better approximate of velocity at each height, a line was fit to the data (mean R^2 = 0.990):

$$v = at + v_i \quad , \quad \textbf{[A6]}$$

where a is the acceleration of the weight, and v_i is the initial velocity of the weight at height h_i. Given the time over which the weight fell from h_i to h_f, the final velocity (v_f) was determined. By substituting these values, obtained for a variety of weights, into **Equation A4**, the moments of inertia of the roller and the wheel were calculated (**Table A**). To completely describe each wheel, the moment of inertia of each tire was added to the moment of inertia of the empty wheel. Each tire was assumed to be a solid torus, with a moment of inertia of

$$I_{torus} = m(R_w{}^2 + \frac{3}{4} r_t{}^2) \quad , \qquad \textbf{[A7]}$$

where R_w is the radius of the wheel (0.264 m) and r_t is the cross-sectional radius of the tire. Since the cross-section of each tire was more elliptical than circular, r_t was replaced by half the tire height times half of the tire width. **Table A** lists the calculated moment of inertia for each tire.

Table A1. Moments of inertia

Object	I [kg·m^2]
Roller	0.870 ± .145
Wheel (empty)	0.068 ± .007
Wheel and tires	0.115 ± .021
Individual tires	
Primo V-Trak	0.0251
Primo Orion	0.0448
KIK Mako	0.0335
Cheng Shin (solid insert)	0.0810
Alshin	0.0496

The moment of inertia of the roller is 4.4% and 6.3% less than the values reported for a similar dynamometer [1]: 0.9096 ± 0.0002 kg-m^2 for the right roller and 0.9281 ± 0.0003 kg-m^2 for the left roller.

References

1. DiGiovine CP, Cooper RA, Boninger ML. Dynamic calibration of a wheelchair dynamometer. J Rehabil Res Dev. 2001;38(1):41–55.