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Wheelchair Batteries: Driving Cycles and Testing[a]

JAMES J. KAUZLARICH, Ph. D.[b]
Professor of Mechanical Engineering

VERNON ULRICH[c]
MARK BRESLER[d]
TED BRUNING[e]
Graduate Research Assistants in Mechanical Engineering

University of Virginia
Rehabilitation Engineering Center
Charlottesville Virginia 22903

ABSTRACT

The battery performance of electric wheelchairs was measured under indoor and outdoor conditions, and simulated driving cycles for these two environments were derived from these tests. Driving cycles were used to bench-test deep discharge wet cell and gel cell lead-acid batteries, nickel-cadmium batteries, and experimental nickel-zinc batteries. Results of this study support the conclusion that deep discharge wet cell lead-acid batteries satisfy wheelchair requirements and are the most economical choice.

The effect of simulated wheelchair controller pulse width modulation on battery discharge compared to d.c. discharge was found to be negligible.

A simple model analogous to Miner's Rule (3) plus results plotted on a Ragone chart of average power versus discharge time were found to correlate the effect of the highly variable actual power requirements of an electric wheelchair. Miner's Rule can predict battery performance for a given driving cycle.

INTRODUCTION

The battery energy storage system is perhaps the most limiting factor in powered wheelchair performance. Deep discharge lead-acid wet cell and gel cell batteries are in common use, and the promise of lighter weight batteries due to higher energy densities with nickel-zinc batteries is under investigation.

Some means of evaluating batteries and interpreting their performance for wheelchairs is needed. An empirical battery model, based on Ragone's equation for power versus discharge time and Miner's Rule[f] for the effect of variable power on discharge time, is presented along with experimental results applicable to powered wheelchairs.

In making a study of battery performance, the experiments can be separated into studies of capacity and of life. Battery capacity is the amount of usable energy a battery can deliver after a full charge, and battery life can be defined as the number of charge-discharge cycles the battery can survive until it can no longer hold a usable charge. The experimental data presented in this report is for battery capacity only. (Results from the literature are cited concerning battery life.)

Battery capacity is usually defined with respect to an operating temperature and a cut-off voltage. At constant temperature a battery's terminal voltage typically drops during use as shown in Figure 1. When the voltage drops below the cut-off voltage at the "knee" of the curve the battery is considered fully discharged.

At the present time the lead-acid battery is the only battery type commercially available at an economical price for use in electric wheelchairs. Nickel-cadmium batteries (Ni-Cd or "Nicad") have been manufactured in sizes applicable for electric wheelchairs, but are

[a] This work was supported by the National Institute for Handicapped Research Grant No. G00-82-00026.

[b] Dr. Kauzlarich is Professor, Department of Mechanical and Aerospace Engineering. He may be addressed at Thornton Hall, University of Virginia, Charlottesville, Virgina 22901. Tel (804) 924-6218 or 977-6370. His other published papers are in areas of friction, wear, lubrication, heat transfer, design, and fluid mechanics. He is the inventor of a welding machine, hydraulic bearings and a medical catheter.

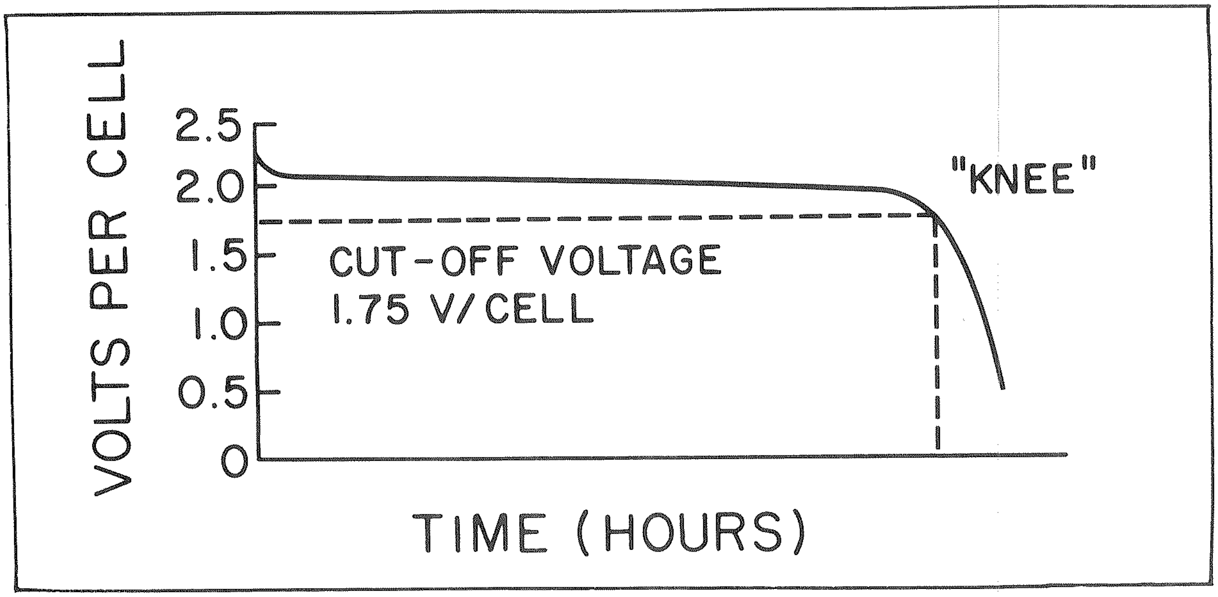
[c] Vernon Ulrich is an engineer with Xerox Corporation, Rochester, New York.

[d] Mark Breseler is Rehabilitation Engineer, O'Donoghue Rehabilitation Institute, Oklahoma City, Oklahoma.

[e] Ted Bruning is an engineer with IBM Corporation, Tucson, Arizona.

[f] The application of "Miner's Rule" for fatigue of metals under varying stresses is discussed under Appendix A of this paper.

FIGURE 1.
Lead/acid battery discharge voltage.



much more expensive than lead-acid. Nickel-zinc batteries have also been manufactured in sizes of interest, but are also much more expensive than lead-acid.

The results of testing these three types of batteries is presented under separate sections. No other types of batteries have been suggested as offering practical battery power for electric wheelchairs, although there are a number of types of batteries being investigated for electric vehicles (1).

THEORY

The capacity of a battery depends upon the geometry of the plates, electrolyte concentration, temperature, rate of discharge, and previous discharge history (2). For a particular battery and temperature the rate of discharge is the significant variable. Previous discharge history is important when the battery is allowed to stand idle for a long time or is discharged at rates and capacities considerably different from normal. A characteristic called battery "memory" may cause a battery to lose capacity if previously discharged only partially for several discharge/charge cycles; several complete discharge/charge cycles may then be necessary to restore the capacity of the battery. (The nickel-zinc and nickel-cadmium batteries tend to show a strong "memory" effect whereas lead-acid batteries show little "memory."[9]

It is usual to express the battery capacity in terms of ampere hours or watt-hours at a given rate of discharge and battery temperature. Battery models have been developed using an equivalent electrical circuit involving capacitors and resistors (3), or by characterizing the performance with empirical equations. The latter approach has been found to be a simple and effective method of analysis and was used for these battery studies. One of the early capacity equations is Peukert's equation relating discharge time T (hours) to the discharge current I (amps), where

$$T = a_1 I^{a_2} \quad [1]$$

Peukert's equation plots as a straight line on log-log paper. Vinal (2, p. 217) finds "the accuracy with which Peukert's formula represents the discharge of these cells (lead-acid) is remarkable."

Another way to represent capacity which has been found to be as simple and effective as using Peukert's equation is to use a Ragone equation (4) relating discharge time T (hours) to discharge power P (watts) similar to eq. [1], as

$$T = C_1 P^{C_2} \quad [2]$$

[9] A letter published in the "Letters to the Editor" section of the Journal of Clinical Engineering, Vol. 8, No. 1, Jan.–March 1983, offers a different view of battery memory. In the letter, the Battery Applications Manager of a major manufacturer advised users of the company's NICAD batteries to "...use them without concern for 'Memory'." He noted, however, that "If you can document a loss of use time in a particular device, leave the device on until the batteries are fully discharged, then recharge. If you have a 'Memory' problem, then your original capacity should return." But his basic statement was that: "Because our batteries on our test do not show memory, we can't comment on whether periodic deep discharge will increase the battery life."

The article which evoked the letter was by Snyder, pages 297–300 in Vol. 7, Oct.–Dec. of the Journal. The writer of the letter was B.G. Merritt, Manager, Battery Applications, Union Carbide Corp., Section A-2, Old Ridgebury Rd., Danbury, CT 06817.

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In addition to equations [1] and/or [2] which were found to fit constant current or constant power discharge data, a method for handling fluctuating discharge rates is required in order to study electric wheelchair battery requirements.

For variable discharge rates the method of Miner's Rule (5) developed for calculating the fatigue life of beams and ball bearings has been applied to batteries in Appendix A, resulting in a simple relation involving the fraction of time $n_i = t_i/T_i$ the battery is discharged at a particular rate. The resulting equation is closely represented by the summation of the fractional energy used in terms of fractional time at each discharge rate of the driving cycle, i.e.,

$$\sum_{i=1}^{k} \frac{t_i}{T_i} = C_M \quad (C_M \approx 1) \qquad [3]$$

Equation [3] is Miner's Rule for calculating battery capacity under fluctuating discharge conditions, where C_M is an empirical constant near unity called Miner's constant, and T_i is given by Ragone's equation (3).

Most electric wheelchairs have pulse-width-modulated (PWM) controllers whose frequency is anywhere from 300 Hz to 1000 Hz (6). To these high frequency discharge pulses, the battery (which is ion-diffusion limited) will respond only on the average. Thus, Ragone's equation was found to apply (see Appendix A) with the battery discharging as if at a constant rate equal to the average discharge power for the high frequency PWM controller. However, when the discharge varies with representative times on the order of seconds or minutes (as in wheelchair operating conditions) Miner's Rule was found to apply. Miner's Rule serves as a simple model for the analysis of wheelchair performance, and the empirical constant can be used to fit the experimental results which then can be used to predict performance under the same conditions as the experiment.

WHEELCHAIR PERFORMANCE EVALUATION AND SIMULATION

Two test routes were used for data collection with an instrumented wheelchair. An indoor route and an outdoor route were selected as representative of the uses of an electrically powered wheelchair for an active user.

The indoor test route shown in Figure 2 is 790 feet (0.241 km) per lap and was intended to simulate continuous use in an office building. The route includes numerous obstacles such as tight elevators and bathrooms, as well as many different floor surfaces ranging from concrete to shag carpet. The wheelchair (described below) was driven continuously over the route at maximum possible speed until the battery was depleted, stopping only to wait for a ride on the elevator. The test required 3.85 hours using a 24-volt, 37 ampere-hour battery, and covered 6.9 miles (11.1 km).

The outdoor test route of 9030 feet (2.75 km) per lap, shown in Figure 3, was intended to simulate a typical route a student or commuter might use to get to class or work. A paved footpath along Massie Road in Charlottesville, Virginia, was selected. In addition to the footpath, the chair was driven around the parking lot of the Judge Advocate General's School. The entire route surface is asphalt paved, but the parking lot has a considerably rougher surface. Rolling hills with grades up to four degrees (1 in 14.3) and an intersection are the only obstacles. The route was repeated until the battery was depleted. The test took 2.74 hours for the 24 volt, 37 ampere-hour battery and covered 9.7 miles (15.6 km).

A Rolls chassis manufactured by Invacare, with a drive system built by General Teleoperators, Inc., was chosen for the testing and is shown in Figure 4. The drive system uses two 1/5-horsepower motors with 15:1 reduction, direct spur gearing. The controller frequency is approximately 400 Hz. The overall empty chair weight is 162.5 lbs. (73.7 kg). With the instrumentation of wattmeter and tape recorder the weight is increased to 192.9 lbs (87.5 kg). A 179.7 lb (81.5 kg) driver was used for all of the tests, making the laden weight of the chair 372.6 lb (169 kg).

A sample of the data taken with the instrumented wheelchair is shown in Figure 5. This particular line of data shows the demand power while going along a level sidewalk (0–1/2 min), making a right-angle turn (1/2–3/4 min), and going along a concrete walk having a 4–5 degree grade (3/4–2 1/2 min). The power demand curve is clearly not very steady, with sharp high peaks due to stop/start action.

A portable wattmeter was designed and built in-house. A circuit diagram is shown in Appendix B. The wattmeter has a gain of 0.005 volts per watt with a range ± 2000 watts. Output is integrated over a 0.3-second time constant to produce a low frequency signal proportional to the average pulse width modulated power.

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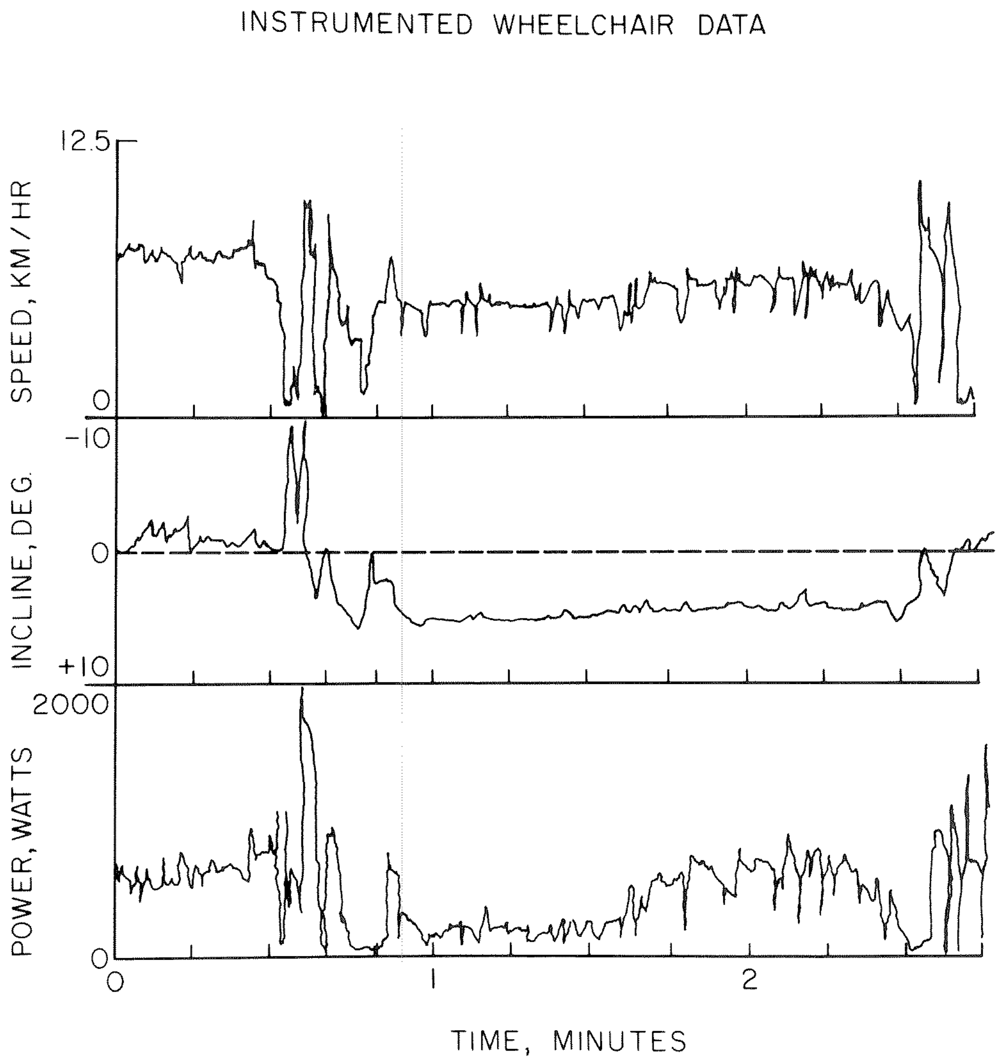


FIGURE 5.
Instrumented wheelchair data.

A TEAC cassette data recorder, Model R-61, was used to record the signal from the wattmeter for eventual playback into a computer analog input. The tape recorder is capable of recording four channels of data and a voice memo. Input voltages were scaled down to a range of 0 to 1.0 volts, corresponding to a range of 0 to 1000 watts.

A commercial wattmeter has also been used and it proved satisfactory for wheelchair testing. The wattmeter is manufactured by EIL Instruments, Inc., Sparks, Maryland, Model WTD-010D, 24 V-AC/DC watt transducer.

For high-frequency PWM circuits a wattmeter will give inaccurate measurements if the frequency response is not good enough. Each of the wattmeters described in this paper was tested on an Acme Electric Corp. programmable solid state load device with a current rise time of .6μs/V up to 2 kHz, and was found accurate to within 6 percent.

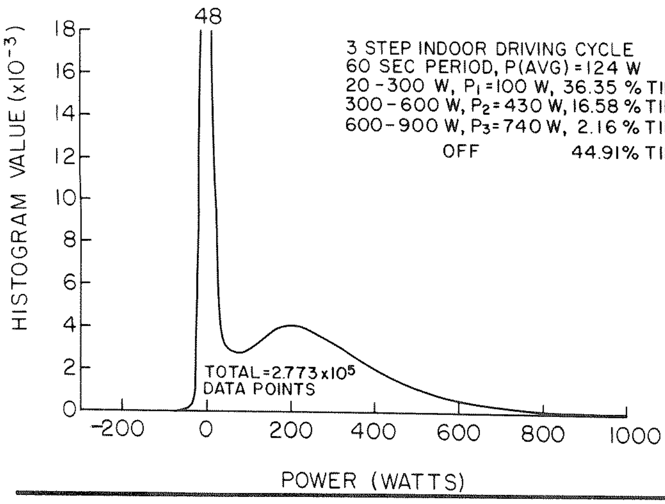


FIGURE 6.
Power histogram, 10 watt step, indoor route.

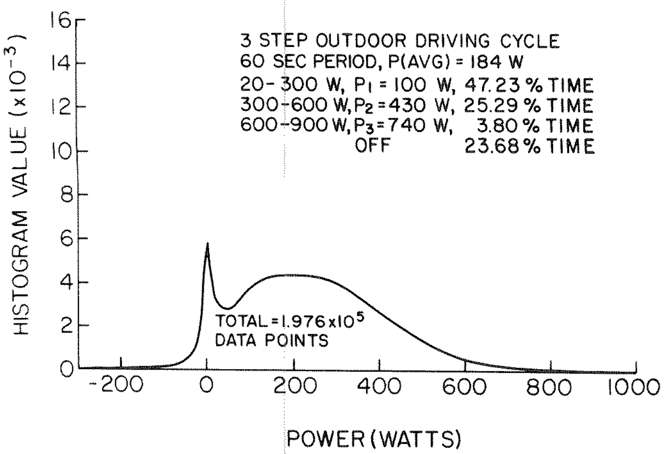


FIGURE 7.
Power histogram, 10 watt step, outdoor route.

RESULTS

Wheelchair Driving Cycles

Histograms of the average battery power data recorded during two tests with the instrumented wheelchair are shown in Figures 6 and 7. Note the peak at 200 watts for both of the tests. From 175 to 200 watts is required to move the chair forward at full speed on a flat and level surface.

Figures 8 and 9 show the relationship between the time of discharge at the average power for the test and the discharge time for the same batteries under constant-power discharge. The square data point symbol on each graph marks the time when the battery voltage first fell to 21 V at load, and the circle marks the time the battery voltage fell to 21 V while driving full speed on a hard level surface.

In each case Miner's Rule was applied to the data using a computer program in order to verify the Rule. Using the Rule predicted a somewhat longer discharge time than measured when using $C_M = 1$. Letting Miner's constant C_M be 0.95 in Figure 8 and 0.97 in Figure 9 will give the same results as the test using 21 V at full speed on the level as the cut-off voltage. (A value of $C_M = 1$ is used for all subsequent calculations.) An example using Miner's Rule is shown later.

For the indoor and outdoor tests, a three-step driving cycle with a 60-second period was selected as a reasonable simulation for bench-testing wheelchair batteries. The cycles are listed in Figures 6 and 7 and are plotted in Figure 10.

The 60-second driving cycle period was established by examining shorter periods of the histogram data, starting with 5 minutes which was the time for one

FIGURE 8.
Battery capacity data, 24-volt 9601 "Diehard" (37 ampere-hour). Indoor test route.

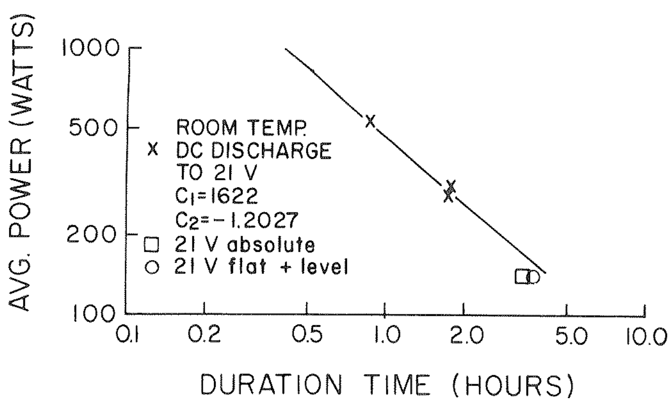
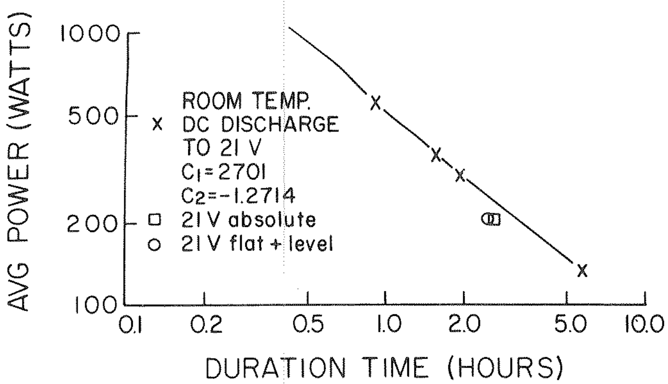


FIGURE 9.
Battery capacity data, 24-volt 9601 "Diehard" (37 ampere-hour). Outdoor test route.



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complete cycle of the indoor test track. Below 60 seconds the resulting histogram began to deviate significantly from the histogram shown in Figure 6.

Battery Bench Tests Using Simulated Driving Cycle

The important discharge variables of temperature, voltage, current, power, and time, were measured for several types of batteries. The tests include direct-current, pulse-width-modulated, and simulated driving cycle discharges.

In setting up the tests the influence of cut-off voltage, temperature, and electrolyte specific gravity were considered as follows.

Electrolyte specific gravity measurements were used to monitor the charge and discharge of lead-acid cells. For lead-acid batteries the concentration of sulfuric acid in the electrolyte changes during charge and discharge in such a way that the specific gravity of the electrolyte can be used as a measure of the state of charge of the cell. A commercially available temperature-compensated electrolyte hydrometer was used to make the majority of the measurements. The hydrometer was checked by measuring the specific gravity of a known solution of sulfuric acid and the results of the measurements were found to be very repeatable. A few measurements of lead-acid cell specific gravities after discharge to a specific cut-off voltage were made with a pycnometer bottle and a precision balance, giving specific gravity results falling between 1.000 and 1.100. The resolution of the commercial hydrometer was 0.25 at the lower end and .010 at the upper end.

Initially, electrolyte temperature measurements at the top of a cell were made throughout all discharges, but no significant temperature changes from the ambient temperature were observed. Therefore, the battery was allowed to reach an equilibrium temperature with the environment and the environment temperature was recorded. All of the testing was done when the ambient air temperature remained about 25 deg C (77 deg F) during the time of the test.

When measuring battery capacity, the end of discharge is generally defined by some arbitrary cut-off voltage. This cut-off voltage is commonly the voltage at the "knee" of the voltage versus time curve, and is rate-dependent (2, p.206). (see Figure 1) For lead-acid batteries, 1.75 volts per cell, or 21 volts for a 24-volt battery (two 12-V batteries in series) was used, and this value was used for all tests. For wheelchair testing of 24-V batteries the time at which 21 volts was first incurred (regardless of the operating conditions) was recorded, but the test was terminated when a 21-volt cut-off voltage was measured under the conditions where the wheelchair was being operated at full speed forward on a hard, level surface.

A. Lead-Acid Batteries

Two 12-V lead-acid batteries connected in series to form a 24-V battery (as currently used in electric wheelchairs) were tested in several types. Among the lead-acid types used by wheelchairs are the automotive wet cell, the deep-discharge wet cell, and gel cell.

At least three factors should be considered in selecting a propulsion battery for electric wheelchairs. These factors are energy density, cost, and cycles of life. The number of cycles of discharge and charge a battery can perform before its capacity degrades below a specified percentage of rated capacity is termed "battery life" and will vary depending on battery construction. For 60 percent depth of discharge (D.O.D) of rated capacity at room temperature, Table 1 gives some representative values of life taken from a battery handbook (1).

In 1982, for a 55 ampere-hour battery, the automotive wet cell and deep discharge wet cell were about the same price ($120 for 24-V), while the gel cell was about twice as expensive. Therefore, on a cost-

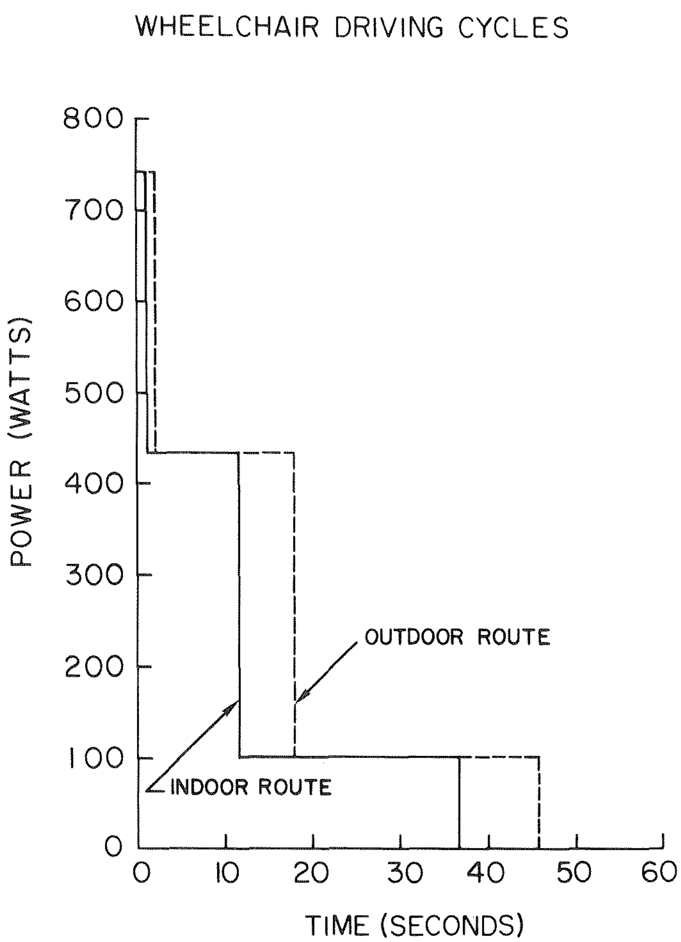

FIGURE 10.
Wheelchair driving cycles.

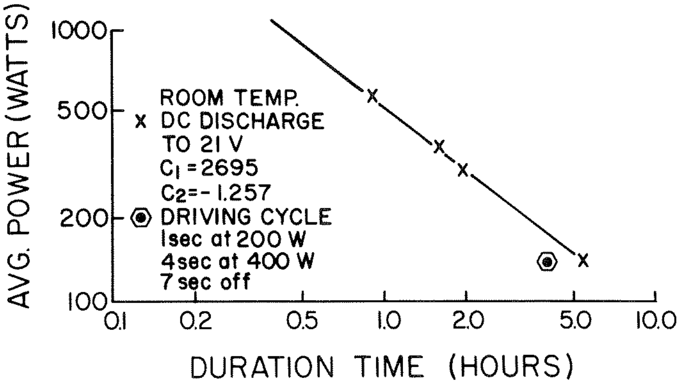

FIGURE 11.
Battery capacity data, 24-volt, 9601 "Diehard" (37 ampere-hour). Driving cycle No. 2.

per-cycle basis (see table 1), the deep discharge wet cell is the best choice for electric wheelchair batteries.

When considering battery performance, the data normally available from the battery manufacturer give direct-current discharge characteristics of the battery. For electric wheelchair applications, the capacity under highly fluctuating discharge rates, and the influence of the pulse-width-modulated controllers used on electric wheelchairs, are of interest. Results showing the effect of simulated electric wheelchair service on the discharge performance of a deep-discharge wet cell and a gel cell are presented in Figures 11 and 12.

Figure 11 gives the d.c. discharge and driving cycle discharge for a Sears deep-discharge wet cell. The listed capacity of 37 ampere hours is for the standard 20-hour discharge rate at 80 deg F (27 deg C). The discharge time for an arbitrary driving cycle (Fig. 1) is plotted at the average discharge power and shows that the discharge time (3.928 hours) to the 21-V cut-off is typically much less than the d.c. discharge time (4.978 hrs.). For the Ragone constants C_1 and C_2 listed on the Figure, and applying Miner's Rule with $C_m = 1.0$, the total discharge time T for the driving cycle is predicted using equations [3] and [2] as follows:

$$\sum_{i=1}^{3} \frac{t_i}{T_i} = \sum_{i=1}^{3} \frac{t_i}{C_1 P_i^{C_2}} = \frac{T}{C_1} \sum_{i=1}^{3} \frac{n_i}{P_i^{C_2}} = 1.0$$

$$\frac{T}{2695} \left[\frac{1/12}{200^{-1.257}} + \frac{4/12}{400^{-1.257}} + \frac{7/12}{0^{-1.257}} \right] = 1.0 \qquad [4]$$

The driving cycle calculation from Equation [4] gives T = 3.924 hours, in good agreement with the experimental value of 3.928 hours. Thus, Miner's Rule wiil predict battery performance for a given driving cycle.

A study of the effect of pulse-width-modulated discharge on the performance of a Globe gel cell is given in Figure 12. The pulse was a square wave with a peak power of 1200 watts. For high frequency pulse width modulation, the discharge time plotted for average power is identical with the d.c. discharge data over the range tested. If Miner's Rule were used to predict the discharge time the prediction would be less than the d.c. discharge time, especially at low discharge rates. It appears that lead-acid batteries, including gel cells, respond on the average to pulse-width-modulated discharge when the cycle period is a small fraction of a second. This result may be due to the slow ion-diffusion rates in the electrolyte, but no further study of the phenomenon was undertaken.

The discharge time for an arbitrary driving cycle is also plotted for the gel cell battery in Fig. 12. Using Miner's Rule with $C_M = 1$ the predicted discharge time is 1.24 hours, while the measured value is 1.12 hours, in good agreement.

Maintenance-free batteries such as the gel cell offer a maintenance advantage that is user-subjective. The gel cell battery will meet aircraft baggage requirements, and can be shipped with the wheelchair aboard most major airlines.

No studies of automotive wet cell batteries were made as there is a considerable body of data available

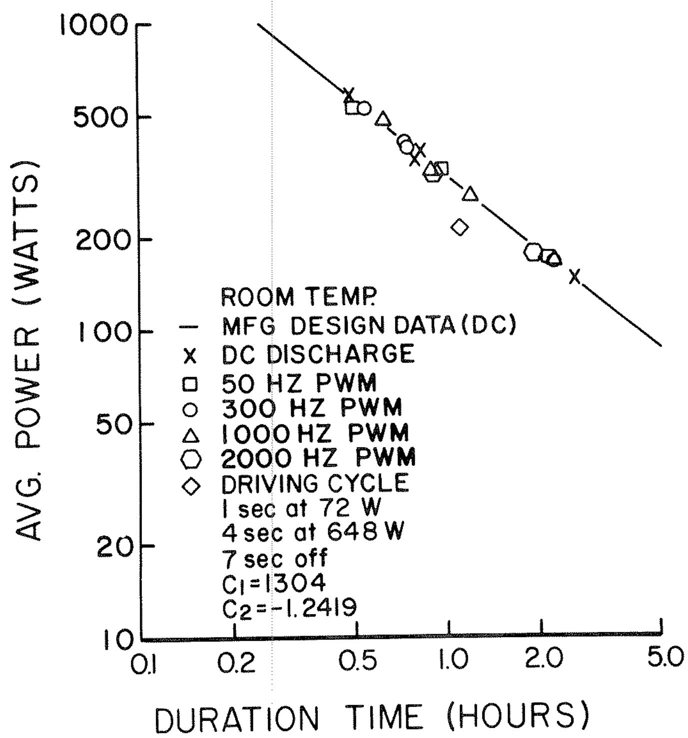

FIGURE 12.
Battery capacity data, 24-volt, U—128 Gel/Cell (28 ampere-hour). Driving cycle No. 1.

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TABLE 1
Lead-acid battery characteristics

	Life at 60% D.O.D. and 80 deg F.	Energy density at 20-hour rate
Type	Cycles	Watt-hr/lb
Automotive wet cell	150–250	12–22
Deep-discharge wet cell (golf cart)	300–500	12–16
Gel cell, deep-discharge	100–300	14–17

for these batteres, e.g., see [5]. Also, because they are not designed for deep discharge, they are not recommended for electric wheelchairs.

To get the most life out of a lead-acid battery, manufacturers recommend that the battery (i) should not be significantly overcharged, and (ii) should be stored in a charged state. (Lead-acid batteries that have been stored in a discharged state may not accept a charge (15). If a battery is not fully discharged during use, its charge/discharge cycle life will be extended; it is reported (8) that the cycle life will be doubled if the depth of discharge is reduced from 100 percent to 60 percent.

A very good discussion of lead-acid battery basics including maintenance, safety, and testing is given by Hudson [9], and is recommended reading for all powered wheelchair users.

B. Nickel-Cadmium Batteries

Nickel-cadmium alkaline batteries have been suggested for electric wheelchairs primarily because of their long cycle-life under deep discharge conditions. Table 1 gives some data taken from references (1, 7, and 10).

The cost of commercial Ni-Cd batteries is about three times that of lead-acid gel cells and about six times that of deep-discharge lead-acid wet cells in production. However, it is predicted (10) that the Ni-Cd offers the possibility of becoming competitive with lead-acid batteries on a cost-per-cycle basis (see Table 1).

Operation of a nickel-cadmium battery involves little change in the 25–35-percent solution of potassium hydroxide used as the electrolyte. Thus, the electrolyte's specific gravity is not a measure of the state of charge—that is measured by the battery voltage at a specific power rate and temperature. For the ampere rate of 20 percent of the 20-hour ampere-hour capacity (C/5) rate and 75 deg F. (24 deg C.), the battery is considered discharged when the voltage is 1 volt per cell, and charged when it is 1.5 volts per cell (1). For a 20-cell, 24-volt battery at 70 deg F. (24 deg C.) the charged to discharged voltage goes from 30 volts to 20 volts.

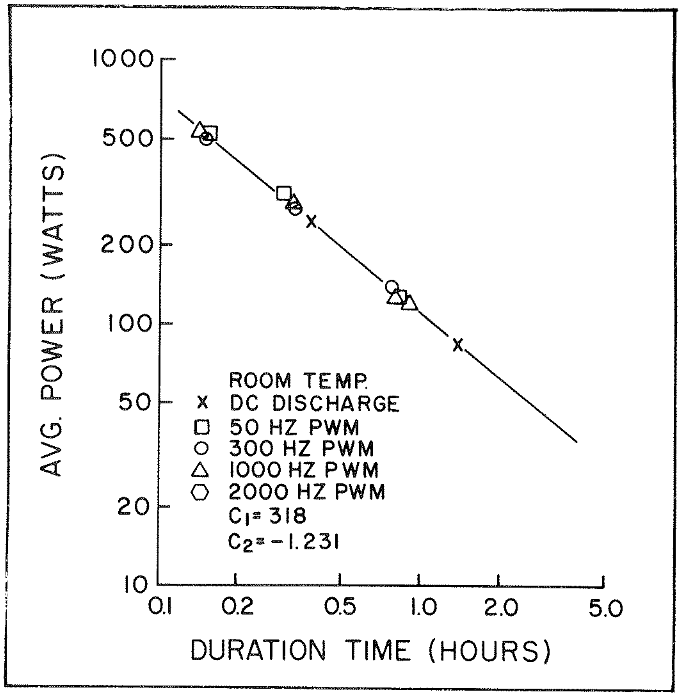

FIGURE 13.
Battery capacity data, 24-volt nickel-cadmium (7.9 ampere-hour).

Figure 13 gives results of a study of d.c. and pulse-width-modulated discharge of a 20-cell, 24-volt nickel-cadmium battery. The experimental results show that the high frequency average-pulse-width discharge rate correlates with the d.c. discharge rate, as was found for the lead-acid battery. The pulse was a square wave with 1200 watts maximum power.

The availability of Ni-Cd batteries in large-capacity sizes of interest for electric wheelchairs is limited.

Nickel-cadmium batteries are relatively maintenance-free, but have a low energy density.

C. Nickel-Zinc Batteries

The nickel-zinc alkaline battery has been recommended for electric wheelchair applications, primarily because of its potential for a high energy-density. However, its poor cycle life for deep-discharge applications, and high cost, have precluded any significant applications to date. Table 1 gives some data from references (1, 11, and 12).

It is estimated (1) that Ni-Zn batteries will be about 25 percent more expensive than deep discharge lead-acid batteries in production lots. On a cost-per-life-cycle basis, nickel-zinc batteries cannot compete with lead-acid batteries; however **one characteristic which may be of practical interest is its good low temperature capacity.** At -40 deg F. (-40 deg C.), where a lead-acid battery has lost almost all of its capacity, a nickel-zinc battery will operate at 95 percent of rated capacity at a current rate equal to 1/5 the capacity (12).

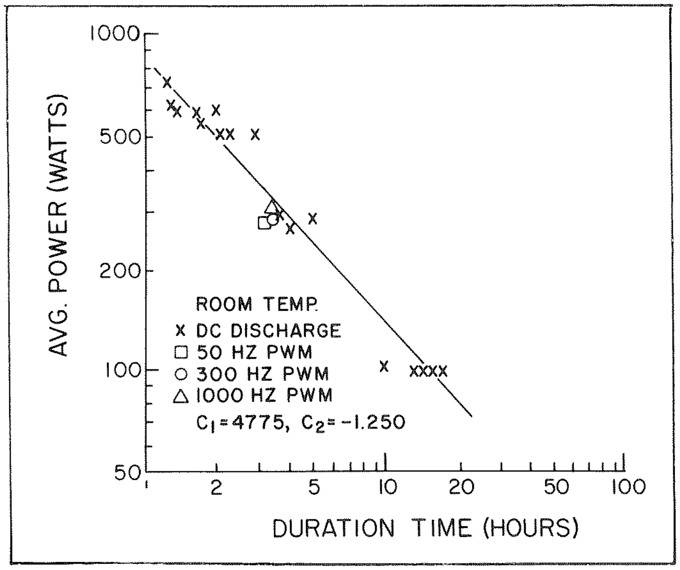

FIGURE 14.
Battery capacity data, 24-volt nickel/zinc (67 ampere-hour).

Figure 14 gives some test results of capacity for a nickel-zinc, 24-volt battery under d.c. and pulse-width-modulated discharge conditions to a 21-volt cut-off at room temperature. Although there is more scatter in the data than in the lead-acid tests, the results are similar. Also, discharge time for pulse width modulation referenced to the average power is closely the same as that for d.c. discharge. The pulse was a square wave with 576 watts of peak power at the frequencies listed in Fig. 14.

Schiffer (11) states, "on a purely cost basis, nickel-zinc batteries will not compete with lead-acid for those jobs which lead-acid can satisfactorily perform."

DISCUSSION AND CONCLUSIONS

For a given battery capacity in ampere-hours, the driving cycles can be used to measure the time the battery will be able to deliver power to a wheelchair. This discharge time can be related to distance traveled. A study of electric wheelchair performance (13) shows that most wheelchairs operate below an efficiency of 45 percent and on level ground under 25 percent. For a 37 ampere-hour battery at 27 deg C. on the indoor route, the range was reported to vary from 3.8 miles (6.1 km) to 7.5 miles (12 km) depending on the wheelchair.

For a given wheelchair, the range was found to vary directly with the ampere-hour capacity of the battery and inversely with the laden weight of the wheelchair. If it is desired to double the range of a wheelchair it would require 2.5 times the capacity for batteries of the same energy density, only 2 times the capacity if the battery energy density were doubled, and no increase in battery capacity if the wheelchair efficiency could be doubled. Thus, several approaches for improving wheelchair performance are indicated.

In the section on lead-acid batteries, it is mentioned that the depth of discharge (D.O.D.) will affect the number of charge/discharge cycles in the life of a battery. The D.O.D. effect thus has an impact on the optimum size (in ampere-hours) of a battery for an electric wheelchair. There is an advantage in selecting a larger-capacity lead-acid battery rather than one of a size that will just suffice for daily use, because for the same amount of daily use the larger-capacity battery will not have to be discharged so deeply, and as we have seen, reducing the depth of discharge can significantly increase the charge/discharge cycle life of the battery. For lead-acid batteries the cost-per-cycle decreases to a minimum at a battery capacity **greater** than required for daily use. For example, if a 20 A-h battery is needed for daily use the cost is .346 $/cycle. However, choosing a 55 A-h battery will reduce the cost to .136 $/cycle, provided that the battery is recharged daily. This effect is a direct result of the fact that the cycle life decreases linearly with depth of discharge. [8].

See Appendix C for details.

In conclusion, this study of battery capacity has resulted in methods for testing batteries for wheelchair applications. The results show that electric wheelchairs operate mainly in the power range of 100 to 1000 watts. The battery cost per charge/discharge cycle is one of the more important practical parameters to consider for electric wheelchair applications, and it has been shown that lead-acid deep discharge wet cells are the best choice among available batteries on this basis.

Two batteries, the nickel-zinc and the nickel-cadmium, offer potential improvements for electric wheelchair applications, but at this time are not economically practical on a cost-per-cycle basis.

Additional battery research and development is needed if significant economical improvements are to be made in electric wheelchair performance due to the battery ■

APPENDIX A

Miner's Rule—A concept of cumulative fatigue damage was proposed by Palmgren in 1924 for ball bearings, and by Miner in 1945 (5) for beams. The hypothesis is called the Palmgren-Miner cycle ratio summation theory, or more popularly, Miner's Rule (14), and is still widely used. Miner's Rule predicts failure of the component when

$$\sum_{i=1}^{k} \frac{n_i}{N_i} > 1 \qquad [5]$$

where, for a given cyclic stress or load amplitude P, n_i is the number of cycles applied and N_i is the number of cycles causing failure at this stress or load amplitude.

For constant-amplitude cyclic stress or load, a power law equation for the number of cycles-to-failure fits the data, where

$$N = C_1 P^{C_2} \qquad [6]$$

This type of equation also fits the data for battery capacity (Ragone's equation) suggesting the applicability of Miner's Rule for fluctuating-discharge capacity calculations.

The assumptions associated with Miner's Rule are best shown by an example calculation. A replot of the discharge equation for the lead-acid battery plotted in Figure 12 is shown in Figure 15, where $T = 1304\,P^{-1.24}$. Assume the battery is discharged at the rates P_3, P_2, and P_1 watts shown in Figure 15 according to the driving cycle shown, i.e., for t_3, t_2, and t_1 hours, and has reached the cut-off voltage for an average power P at time T hours. The energy discharged is:

$$P_3 n_3 T_3 + P_2 n_2 T_2 + P_1 n_1 T_1 = \bar{P}\,T \qquad [7]$$

where

$$n_1 = t_1/T_1, \text{ etc.} \qquad [8]$$

thus,

$$\frac{P_3 T_3 n_3}{\bar{P}\,T} + \frac{P_2 T_2 n_2}{\bar{P}\,T} + \frac{P_1 T_1 n_1}{\bar{P}\,T} = 1 \qquad [9]$$

Applying Ragone's equation

$$T = C_1 P^{C_2} \qquad [10]$$

get:

$$n_3 \left[\frac{P_3}{\bar{P}}\right]^{1+C_2} + n_2 \left[\frac{P_2}{\bar{P}}\right]^{1+C_2} + n_1 \left[\frac{P_1}{\bar{P}}\right]^{1+C_2} = 1 \qquad [11]$$

For the battery of Figure 15, $C_2 = -1.24$. If it is assumed that $C_2 = -1$, Eq. [11] reduces to Miner's Rule, where

$$n_3 + n_2 + n_1 = 1 \qquad [12]$$

or, in general

$$\sum_{i=1}^{k} \frac{t_i}{T_i} = 1 \qquad [13]$$

The Ragone equation for Miner's Rule is $T^1 = 334\,p^{-1.0}$, and the plot shows the approximation associated with using the Miner's Rule assumption.

The average power is calculated from

$$\bar{P} = \frac{1}{T} \sum_i P_i t_i \qquad [14]$$

The total discharge time T for a fluctuating discharge rate is calculated from:

$$\frac{T}{C_1} \sum_i \frac{t_i/T_i}{P_i^{C_2}} = C_M \; (C_M \approx 1) \qquad [15]$$

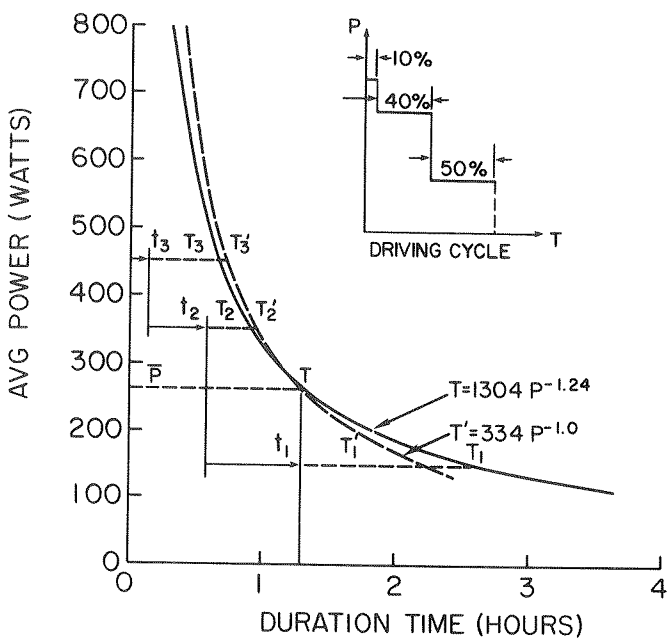

FIGURE 15.
Battery capacity and driving cycle, 24-volt lead/acid (37 ampere-hour), 77 deg. F.

APPENDIX B

Electronic Wattmeter Design—Figure 16 is the wattmeter circuit. Basically the battery voltage is divided across a resistor and multiplied by the amplified current shunt voltage to produce a signal which is proportional to the average pulse-width-modulated power of the wheelchair battery. The sensitivity is adjusted to 200 watts/volt, and the output is integrated over a 0.3 second time constant. The range is ± 2000 watts. The warmup time is about 15 minutes. The current drain is about 20 mA, with 15 hours of operation possible using NEDA 1603 batteries.

Calibration is as follows:

1. After power is on for 15 minutes, measure the supply voltage. The voltage should read + 18 volts and − 18 volts with respect to ground, although any voltage level in the range of 15 to 18 volts is acceptable.
2. Connect 1 volt from the + **Battery Voltage** terminal to ground and adjust the + VOLTS 2k Pot. for 50 mV from its wiper to ground. Move the 1-volt source to the −**Battery Voltage** terminal to ground and adjust the −VOLTS 2k Pot. for 50 mV from its wiper to ground. Remove the 1 volt source.
3. Connect 24 volts across the **Battery Voltage** terminals for the rest of the calibration.
4. Measure the voltage from the final output of the **Current Signal Amplifier** (pin 4 of the 534 I.C.) to ground. Adjust the AMP AMP ZERO 5k Pot. in the **Zeroing Voltage Regulator** for 0 volts. Then measure the voltage at the **Watts Out** terminals and adjust the WATTS ZERO 1k Pot. in the **Zeroing Voltage Regulator** for 0 volts.
5. Set the **Gain Calibration Voltage** switch to On and the Cal/Run switch to Cal. Connect 1 volt to the **Gain Calibration Voltage** input, and adjust the WATTS CAL. 5k Pot. for 25mV from its wiper to ground.
6. Measure the voltage at the **Watts Out** terminals and adjust the WATTS GAIN 25k Pot. in the **Watt Signal Amplifier** until the reading is 6 volts (200 watts/volt).
7. Repeat the calibration for fine adjustment.
8. Connect **Current In** and **Battery Voltage** terminals to the battery to be tested, set the **Gain Calibration Voltage** switch to Off, set the Cal/Run switch to Run, and proceed with the test. The shunt is 100A, 50mV.

Using a multimeter with an accuracy of ±0.1 millivolts for calibration gave results at the 95 percent confidence limits of ±6 watts over the range of the wattmeter.

APPENDIX C

Depth of Discharge Effect—The discharge/charge cycle life of lead-acid batteries decreases linearly with the depth of discharge (D.O.D., percent) for an optimum charging voltage of 2.45 V/cell and a 16-hour charge, over a D.O.D. of 60-to-100 percent [8]. The equation is

$$\text{Cycles} = 1260 - 10 \times \text{D.O.D.} \qquad [16]$$

For deep discharge lead-acid batteries the cost in $ for a 24 V battery (two 12-V in series) is found to vary linearly with capacity (C, ampere-hours), where (1982):

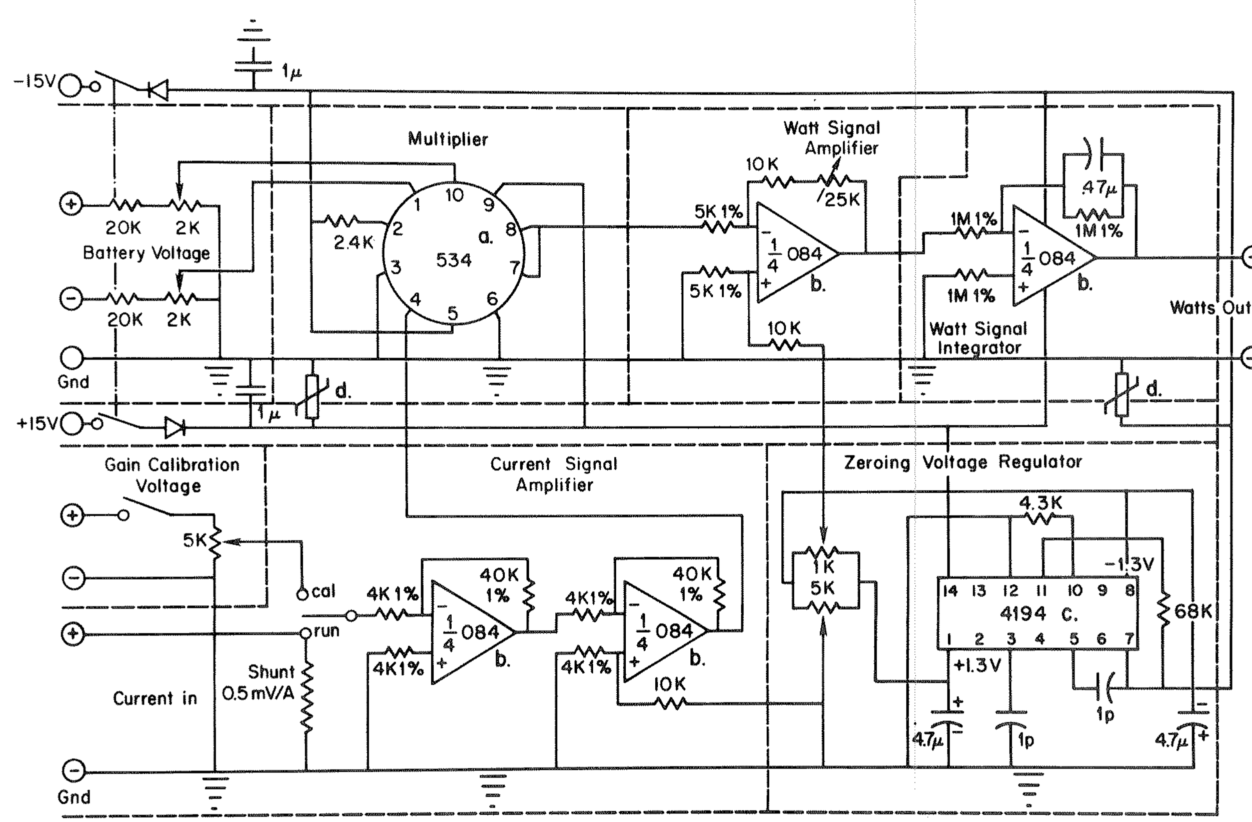

FIGURE 16.
Wattmeter circuit.
a. Analog Devices. b. Texas Instruments. c. Raytheon. d. General Electric.

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$$\$ = 72 + .9 \times C \qquad [17]$$

The depth of discharge is defined by

$$\text{D.O.D.} = \frac{C_o}{C} \times 100, \text{ percent } C \geqslant C_o \qquad [18]$$

Where, C_o is a reference capacity (in ampere-hours) required per day, and C is the actual battery capacity installed in the wheelchair.

The equation for cost-per-cycle, based on Eqs [16, 17, & 18] is

$$\frac{\$}{\text{Cycle}} = \frac{(72 + .9 \times C) \times C}{1260\ C - 1000\ C_o} \qquad [19]$$

This quadratic equation has a minimum value at

$$C_{opt.} = .794 \times C_o + \sqrt{.63 \times C_o^2 + 63.5 \times C_o} \qquad [20]$$

For $C_o = 20$ A-h, the capacity required for daily use, $C_{opt.} = 55$ A-h from Eq. [20]. The cost-per-cycle calculated from Eq. [19] is .346 \$/cycle for C=20 A-h, and .136 \$/cycle for C=55 A-h. Thus, choosing a battery with a larger capacity than needed for daily use will significantly reduce the cost per cycle of charge/discharge over the life of the battery. (This calculation assumes that the battery is recharged daily). The effect of battery weight was neglected, as it was on the order of 10%.

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