Properties of an adaptive feedback equalization algorithm

A. Maynard Engebretson, DSc and Marilyn French-St. George, PhD
Central Institute for the Deaf, St. Louis, MO 63110

Editor’s Note:
This is the first of two related papers in this issue on the subject of adaptive feedback equalization in digital hearing aids. This article describes an adaptive algorithm that estimates and tracks the characteristic of the hearing aid feedback path.

The second article, “Behavioral Assessment of Adaptive Feedback Equalization in a Digital Hearing Aid,” by Marilyn French-St. George, et al., describes the results of behavioral testing of the algorithm in subjects with hearing impairment.

Abstract—This paper describes a new approach to feedback equalization for hearing aids. The method involves the use of an adaptive algorithm that estimates and tracks the characteristic of the hearing aid feedback path. The algorithm is described and the results of simulation studies and bench testing are presented.

Key words: acoustic feedback, adaptive feedback equalization, feedback instability, new adaptive equalization algorithm.

INTRODUCTION

Feedback instability is a problem with hearing aids that often results in performance degradation and reduced benefit for the listener with hearing impairment. Feedback instability reduces battery life. It limits the gain that can be prescribed.

Furthermore, it is often a source of embarrassment for the wearer when the hearing aid breaks into a loud oscillation at inappropriate times. The problem is most serious with high-power hearing aids that develop high gains and with in-the-ear packages where acoustical and mechanical isolation between receiver and microphone is limited.

The theory of feedback instability is well understood (1). The basic concept of feedback is illustrated in Figure 1a. If the feedback signal, Z(f), is out of phase with the input, X(f), negative feedback results that is often used in the design of systems to stabilize them and make the systems less sensitive to component variation. If the feedback signal is in phase and greater in amplitude than the original input signal [the loop gain, G(f)*A(f), is greater than 1], the system becomes regenerative and unstable. In the case of acoustic feedback, the phase of the feedback signal is a function of frequency. Therefore, the feedback signal will be in phase and out of phase at several frequencies within the bandwidth of the system, and if the loop gain is
greater than 1 at these frequencies, the acoustic system will oscillate. Acoustic systems can oscillate at multiple frequencies, depending on the phase and amplitude relations of the loop gain with respect to frequency.

It is also possible for a feedback system to perform poorly but yet be stable in the sense of the above discussion. This can occur if the loop gain is in phase but at a gain that is slightly less than 1. In this case, the system will be stable but underdamped and will exhibit aberrant resonant peaks that are undesirable. Since many people who wear hearing aids adjust the volume control to just below the point where the hearing aid oscillates, it is likely that many are experiencing a highly resonant, underdamped characteristic.

The problem of hearing aid instability is difficult to solve. First of all, practical considerations are such that the acoustic output and input of the hearing aid cannot be well isolated. People often prefer small, in-the-ear hearing aids for reasons of aesthetics and convenience. Others prefer behind-the-ear hearing aids with earmolds that are vented or open to provide greater comfort. Tight-fitting, unvented earmolds are uncomfortable, collect moisture, and exhibit the occlusion effect wherein the wearer hears his/her own voice. With open earmolds, the magnitude of the feedback path is often close to unity and the phase varies on the order of 180 degrees per 1,000 Hz. Therefore, we are forced to deal with basically an unstable system when we try to achieve acoustic gains of 20 to 40 dB over a bandwidth of 8 kHz or more.

Egolf (2) has reviewed the acoustic feedback literature and describes a number of potential methods for stabilizing hearing aids and detecting instability. These methods include use of tunable notch filters, frequency shifting, phase shifting, and frequency modulation for stabilization and correlation and phase-lock-loop methods for detecting oscillation. Each of these methods seems to have a number of deficiencies as applied to hearing aids. The notch filter approach requires that the offending frequency of instability is known. However, if the frequency is known, improvements in gain of 7-17 dB can be achieved. The other three methods provide only modest improvement in gain margin (6 dB) and introduce perceptible distortion. A different approach that is more attractive is active equalization, which is shown in Figure 1b. The idea here is to simulate the feedback path of the hearing aid with an electronic filter, E(f), that is connected in parallel with the feedback path, A(f), to cancel the signal that feeds back at the input to the hearing aid, Z(f). This approach works well for time-invariant systems. Egolf reported that, with considerable fine-tuning, gain margins of 15 to 20 dB were achieved in the laboratory. However, since the
feedback path of the hearing aid is constantly changing, in order to get good cancellation the equalization filter must adapt to accommodate those changes, as shown in Figure 1c.

Studies of adaptive equalization filters with wearable hearing aid systems have been reported by Dyrlund and Bisgaard (3) and Engebretson et al. (4,5,6). Both groups of investigators have obtained similar results, that stable gain margins of hearing aids can be improved by 10 to 15 dB with adaptive equalization. Not only does adaptive equalization suppress oscillation, but it equalizes the underdamped, non-oscillatory feedback condition that often occurs at high gains and tends to degrade hearing aid performance.

The purpose of this paper is to describe the performance and limitations of one such adaptive equalization algorithm that has been implemented in digital form and that is the basis for the behavioral study reported in another paper in this issue (7). The algorithm is incorporated into a wearable version of the CID digital hearing aid (8) and differs in a number of ways from other implementations. First, the digital hearing aid uses logarithmic arithmetic to simplify the very large scale integrated (VLSI) circuitry. Second, the coefficients of the adaptive filter are implemented as up-down counters, which reduces the complexity of the VLSI circuitry further. The goal here is to achieve a circuit design of modest complexity that is practical to implement in the form of a small semiconductor chip that is compatible with the constraints of an ear-level hearing aid package without compromising the functionality of the digital hearing aid. Therefore, considerable attention has been given to developing an adaptive algorithm that is optimal with regard to performance, power consumption, and size.

The remainder of the paper is organized as follows. First, the feedback characteristics of a typical hearing aid are examined. Second, the model of feedback equalization is described. Simulation studies of the model are presented to demonstrate the performance of the binary, logarithmic adaptive algorithm. Third, results of bench tests using a KEMAR mannequin are presented. The relation between filter length and degree of equalization is examined. Results of tests of the system in subjects with hearing impairment are presented in the related article in this issue (7).

**METHODS/RESULTS**

**Nature of Feedback Path**

The feedback path of a hearing aid includes both mechanical and acoustical coupling between the receiver and microphone. However, the acoustic leakage path is the primary one. This is illustrated in the measurements of Figure 2, which were obtained with an experimental in-the-ear module containing a typical hearing aid receiver and microphone. The measurements were made on a KEMAR mannequin with a Zwislocki coupler using instrumentation amplifiers and phase meter. The receiver was excited with a 70 mV rms voltage drive, and the microphone preamplifier signal was measured with phase referenced to the receiver voltage. The condition shown in the figure is for a relatively loose-fitting ear module.

These results are typical. The feedback coupling is poor at low and high frequencies and one or more peaks representing resonances lie in between. The phase is approximately linear with respect to frequency and represents a propagation delay of about one-half millisecond, which is equivalent to an acoustic path length of 16 cm. Since the dimensions of the ear module are much smaller than 16 cm, the phase of the feedback characteristic is not dominated by acoustic delays. Instead, phase is primarily determined by the delay of the receiver. The amplitude of the feedback signal changes with tightness of fit between the ear module and ear. The

![Figure 2. Measured feedback characteristic of typical ear module. Typical are the multiple, low-Q resonances and large phase angles.](image-url)
better the seal, the lower the amplitude. However, the phase remains about the same. Objects and surfaces in the vicinity of the ear modify the resonant peaks in both amplitude and frequency by creating standing waves. For example, the presence of a hat brim near the ear can increase the amplitude of the feedback signal and is known to cause hearing aids to oscillate.

The results described above are consistent with the models of Egolf et al. (9) and Kates (10). However, they differ somewhat. Egolf et al. analyzed an eyeglass-style hearing aid but did not incorporate a hearing aid receiver in the mathematical model or include an ear canal. They modeled the transfer characteristic from a point near the external opening of the vent of the earmold to the microphone port located on the eyeglass frame. Therefore, the primary delay in their model is acoustic and the phase shifts are considerably less than what we observed. Kates included a model of the receiver, ear canal, and vent in his simulation. However, the simulation exhibited a sharp, single resonance in the transfer function of the feedback path at about 7 kHz, which is in contrast with our observations of relatively low-Q, multiple resonances at much lower frequencies. The reason for pointing out these differences is that they have an impact on the choice of parameters of the equalization mode. For example, the low-Q resonances observed in our hearing aid system require a shorter equalization filter than the high-Q resonance of the Kates model to achieve the same degree of cancellation. We have observed, however, that greater delays will require longer filters.

Feedback Equalization Model

It appears that the feedback path, whether it be a leak around the earmold, through a vent, or both, can be modeled as a filter. This is shown in Figure 3 where the external feedback path, which is a composite of all sources of feedback, is represented by $H_f$. The feedback equalization filter, $H_e$, is connected between the output of the aid and the input. The output of the equalization filter is subtracted from the input signal to cancel the contribution from the external feedback path. The error signal, $E$, is the difference between the two paths and is used to adaptively adjust the coefficients to minimize this difference in a least-mean-square sense (11). A random noise source, $N$, which is uncorrelated with other signals in the system, is included to excite the system when signals are small. The noise source typically is set to a level that is low enough to be unobtrusive to the listener. Other sources of random noise also excite the system and serve the same purpose as is described below.

In the diagram in Figure 3, $H_m$ and $H_r$ represent the transfer characteristics of the microphone and receiver, respectively. $H_m$ includes the analog-to-digital converter transfer characteristic and $H_r$ includes the digital-to-analog converter transfer characteristic. $H_f$ and $H_e$ are as described above and $H$ is the desired prescriptive frequency-gain function of the hearing aid. The signals $X'$ and $Y'$ represent the acoustic input and output of the hearing aid, respectively, and $X$ and $Y$ represent the digital equivalent of these signals, including the transfer characteristics of the microphone, preamplifier, ADC, DAC, power amplifier, and receiver.

The adaptive algorithm that is used to adjust the coefficients of $H_e$ is based on the LMS algorithm (11). The expression

$$u(n) = \Sigma c_i y(n-i)$$

is a filtered version of the output signal, $y(n)$, that is an estimate of the external feedback path. The cancellation error is:

$$e(n) = u(n) - v(n)$$

where $v(n)$ represents the external feedback signal that the adaptive filter is attempting to cancel. The
mean of the squared error, $e^2(n)$, has a unique minimum with respect to the coefficients of the equalization filter, $H_e$. The coefficients are adaptively driven toward this minimum by using the gradient of the error to determine the direction of steepest descent. An algorithm for adjusting the coefficients that requires no direct calculation of the gradient and no multiplications is given by:

$$C_k(n + 1) = C_k(n) + \lambda \text{sign}[y(n - k) e(n)]$$

where $\lambda$ is a constant that is a power of 2. Therefore, updating the coefficients is equivalent to incrementing or decrementing the coefficient register, depending on the value of the sign function. The sign function is a simple exclusive-or function of the sign bits of $e(n)$ and $y(n - k)$. Typically, $\lambda$ is chosen to be 1/64 the least significant bit of the coefficient by extending the coefficient registers six bits below the least significant bit. This corresponds to a value that is 1/128 dB. The added least significant six-bits accumulates an average that reduces the variance of the estimation. The resulting coefficient value can be considered to be a stochastic average that is related to the correlation between the cancellation error and the coefficient. Since the values of the coefficients are in log units, incrementing and decrementing them is equivalent to multiplying and dividing the coefficient values by a constant percent in the linear sense. Although it may not be obvious, this does not change the robustness and stability of the basic LMS method.

**Simulation Study of Adaptive Algorithm**

The behavior of the algorithm is illustrated in Figure 4 for a simulated open-loop condition ($H = 0$) For this illustration, the external feedback signal, $v(n)$, is derived from the model expression:

$$v(n) = 10 y(n - 2) - 5 y(n - 4)$$

where $y(n)$ is a pseudo-random sequence, $p(n)$, that also serves as the input to the adaptive filter. It can be seen that the coefficients follow a logarithmic path in adapting to the model values and that once the correct values are reached the algorithm randomly dithers around the least significant bits of the coefficients. The sign function simplification of the LMS algorithm results in a slow rate of adaptation. However, this is desirable in many applications. The simplified algorithm has other desirable characteristics. For example, since the sign function is either zero (decrement the coefficient) or one (increment the coefficient), no dead zone occurs, as the error becomes exceedingly small, that will cause a coefficient tracking offset error. In addition, the coefficients can be updated in any order, singly or together, at any sampling rate up to the sampling rate of the system. Therefore, there are a number of possibilities for optimizing the implementation and for varying the rate of adaptation.

We were concerned initially that the algorithm would not converge properly if the system started in an oscillatory state. However, this is not a problem. Figure 4 also illustrates the behavior of the algorithm for an oscillatory closed-loop condition ($H = 1$). It can be seen that when the equalization filter is initialized to zero, the rms error quickly grows to a maximum as the system begins to oscillate. Although the time required for equalization is greater when the system is oscillatory, it can be seen that the coefficients eventually reach their desired values and the system becomes stabilized.

In either the open-loop or closed-loop case, once the final state of equalization is reached, the coefficient values dither randomly about the desired values with an error that is proportional to the coefficient value. This generates noise that is uniformly distributed across all frequencies and limits the degree of cancellation possible. The least significant bit of the coefficients is equivalent to 0.5 dB in our implementation, or 6 percent. Assuming that the coefficient error is uniformly distributed between $\pm 3$ percent, the rms error is equal to about 0.9 percent. Therefore, the coefficient noise will be on the order of 40 dB below the signal level.

It has been mentioned above that a pseudo-random probe noise is inserted at the output of the hearing aid to serve as a common source of low-level sound for exciting the feedback path and the equalization filter. However, it has been found that other sources within the hearing aid generate an appropriate wide-band noise that serves the same purpose. Furthermore, these sources generate noise that is proportional to the signal level. Note that the Dyrlund and Bisgaard (3) implementation requires a separate circuit to adjust the level of the probe noise so that it is about 30 dB below the signal level. With log encoding, the quantizing noise is white and is 35 dB below the level of the encoded signal. Arithmetic roundoff noise and the dithering of the coefficients adds additional noise at levels about 40 dB below
Figure 4.
Simulation results of the log-binary-LMS adaptive algorithm for a simple model showing typical coefficient behavior for open-loop and closed-loop conditions. (a) Time course of coefficients for open-loop case; (b) time course of rms cancellation error for open-loop case; (c) time course of coefficients for closed-loop case; (d) time course of rms cancellation error for closed-loop case.

There are several factors that limit the amount of additional stable gain that can be achieved with the feedback equalization algorithm. The first factor is the degree of cancellation that can be achieved. Because of the choice of log base and the number of bits used to represent the coefficient values, the coefficient estimation error is on the order of 0.9 percent. The rms error between the equalization filter and the feedback path is also 0.9 percent. Therefore, the maximum gain margin that can be expected will be about 40 dB due to this source of error alone. Another factor that limits gain margin is the presence in the system of narrow-band or periodic signals that have long autocorrelation functions. These types of signals will cause the adaptive algorithm to deviate from the estimate of the feedback path. This effect is reduced by delays through the digital filters of the hearing aid, which move the offending autocorrelation terms to later lag products. Also, because of the slow adaptation rates that are used (several seconds), only periodic external signals that persist will upset the equalized state of the system.
Figure 5.
Bench-test measurements of equalization filter after it has reached a steady state for two conditions of acoustic leakage. (a) Impulse response for loose earmold condition; (b) frequency response for loose earmold condition; (c) impulse response for tightly fitting earmold with vent condition; (d) frequency response for tightly fitting earmold with vent condition.

Bench Tests with KEMAR Mannequin

Bench testing of the adaptive algorithm on a KEMAR mannequin has been extensive. Typically, a Macintosh computer is used as a host system, and programming of the digital hearing aid is accomplished via a serial port. It is also possible to upload the equalization filter coefficients to the host computer via the serial port so that they can be observed. Figure 5 illustrates the impulse response of the equalization filter for two conditions of acoustic leakage with a KEMAR mannequin test setup after a steady state of equalization has been reached. The impulse response (Figure 5a) represents an estimate of the external feedback characteristic of the hearing aid. The frequency response of the equalization filter (Figure 5b) is obtained by taking the Fourier transform of the impulse response. The result in this figure can be compared with the direct measurement of the feedback characteristic of Figure 2. Results with a tighter fit and a vent are shown in Figure 5c and Figure 5d. Most of the delay before the start of the impulse responses in the figures (each tap corresponds to a 60 μs delay) is due to the delays through the receiver, ADC, and DAC.

The equalization filter has to be long enough to span the impulse response of the acoustic feedback path. The relationship between the length of the equalization filter and gain margin has also been studied and typical results are shown in Figure 6. Each curve represents the greatest gain that could be
achieved without oscillation for an initial delay of 8 samples and an equalization filter with 16, 32, 48, and 64 taps. As can be seen, a total span including initial delay and filter of 56 samples is required to achieve a maximum stable gain. This corresponds to a delay of 3.36 ms, which can be compared with the impulse responses of Figure 5. It should be noted that the gain margin, which is the difference in achievable stable gain with and without feedback equalization, is about 20 dB. The adaptive behavior of the system at the limit of maximum achievable gain is shown in Figure 7. The curve represents the error between the external feedback path and the internal equalization filter. When starting from zero, the system requires about 1.5 seconds before approaching an equalized state and then an additional second while each of the coefficients reaches its final state. These results are not unlike the simulation studies described earlier in the paper.

DISCUSSION

The feedback equalization method described above appears to be a viable solution to the problem of hearing aid instability. The algorithm behaves robustly and is suitable for implementation in the current generation of hearing aids. Additional usable gains of 10 to 15 dB can be achieved in practice, which corresponds to an additional population of hearing-impaired with 20–30 dB greater hearing loss that can be helped. In addition, open earmolds, which provide greater comfort, can be used more frequently with moderate hearing loss. We estimate that the algorithm can be implemented in the form of a small, low-voltage circuit that will require substantially less than 1 mW of power.

We recognize that this is one of the first attempts to apply principles of adaptive active cancellation to hearing aids. We hope that, as with other engineering endeavors, when more designers begin to apply their skills to the problem, improved algorithms will result that will extend performance and provide even greater benefit for the listener with hearing impairment.

ACKNOWLEDGMENTS

The authors would like to acknowledge the significant contribution of Michael O'Connell and Arnold Heidbreder in developing the hardware and software for the digital hearing aid and fitting system and in bench testing and calibration. This work was supported, in part, by the Rehabilitation Research and Development Service of the Department of Veterans Affairs and the National Aeronautics and Space Administration.
REFERENCES


