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Distributed random electrical neuromuscular stimulation: Effects of the inter-stimulus interval statistics on the EMG spectrum and frequency parameters

Yuan-Ting Zhang, PhD; Philip A. Parker, PhD; W. Herzog, PhD; A. Guimaraes, PhD
Departments of Electrical & Computer Engineering, Biomechanics, and Human Performance Laboratory, University of Calgary, Calgary, AB T2N 1N4, Canada; Department of Electrical Engineering, University of New Brunswick, Fredericton, NB E3B 5A3, Canada.

Abstract—An electrophysiological approach was used to study a distributed random electrical neuromuscular stimulation (ENMS) scheme in which a probability density is assigned to the inter-stimulus intervals (ISI) of the stimuli. One of the objectives of using ENMS techniques in the study of skeletal muscles is to obtain information about the electrical, physiological, and mechanical properties of muscles in a near-physiological situation under a well-controlled experimental design in which problems related to the uncertainty of firing patterns of the central nervous system and physiological interference are avoided. In particular, ISI with a Gaussian density were varied in mean rate, standard deviation (SD), and coefficient of variation. The influence of varying ISI, and the interaction of the ISI statistics with compound motor unit action potentials (CMUAP) on EMG power spectra and their frequency parameters, was assessed theoretically using a mathematical model which is similar to that of EMG signal generation in the electrophysiological case. In order to quantify the effects of ISI statistics on the EMG spectrum, the median frequency was calculated as a function of stimulation rate using analytical expressions for various values of the coefficients of a Gaussian ISI variation. The results obtained suggest that 1) the interaction between ISI statistics and the shape of the CMUAP plays a major role in determining the EMG spectrum; 2) the median frequencies (MF) determined from EMG spectra tend to increase with increasing mean rates of stimulation for a given CMUAP. The rate of increase of the MF depends on the coefficient of the ISI variation; 3) the EMG spectra of random electrically stimulated muscle show peaks at the mean rate of stimulation, and multiples of it, when the coefficient of variation of ISI is small. These peaks decrease in magnitude with increasing coefficients of variation of ISI; and, 4) a variation in the ISI should be introduced in the ENMS, when a reproduction of 'normal' EMG spectra is needed. These results are consistent with those reported for voluntary contraction of skeletal muscles.

Key words: *electrical neuromuscular stimulation, electromyography (EMG), spectral analysis.*

Address all correspondence and requests for reprints to: Dr. Yuan-Ting Zhang, Department of Electronic Engineering, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong; Email: ytzhang@ee.cuhk.hk.

P.A. Parker is with the Department of Electrical Engineering, University of New Brunswick, Fredericton, NB, Canada; W. Herzog and A. Guimaraes are with Biomechanics, and Human Performance Laboratory, University of Calgary, Calgary, AB, Canada.

INTRODUCTION

Electrical neuromuscular stimulation (ENMS) has become a common and important technique to study muscle activities, including electromyographic signals (EMG), biomechanical outputs, such as muscular force and vibromyographic (VMG) events (known also as muscle sounds), and their relationships (1–7). One of the objectives of using ENMS techniques in the study of skeletal muscles is to obtain information about the electrical, physiological, and mechanical properties of muscles in a near-physiological situation under a well-controlled experimental design in which problems related to the uncertainty of firing patterns of the central nervous system and physiological interference are avoided. For studies with this objective, it is necessary to select appropriate stimulation parameters and strategies, in accordance with findings from research in electrophysiology.

A literature review (8–13) of electrophysiological studies of muscle identifies the following features: 1) force production in skeletal muscle is controlled by the number of motor units and the rate of firing of each unit; 2) EMG signals from active units are uncorrelated during low and medium voluntary effort, but may become synchronized at high levels of effort; and, 3) firing of alpha motor neurons is a random process, in the sense that the time interval between two successive spikes, that is, the interspike interval (ISI), is a random variable described by a probability density function (pdf). These random firing patterns of motor units have been widely studied during voluntary contraction in human skeletal muscles (9,14–17). Effects of the firing statistics, especially the mean firing rate, on the EMG spectrum have been described using mathematical models and experiments during tasks of moderate, nonfatiguing, constant effort isometric contractions (15,18–22). These findings suggest that the choice of the ISI statistics, in a study involving ENMS, may influence the median and mean frequencies of the EMG spectrum, two parameters commonly used to assess muscle properties during tests which involve voluntary contractions (10,14,23).

Because of technical difficulties and a lack of adequate physiological information as well as because of the research questions being asked, early research involving ENMS often was not aimed at representing the actual physiological situation. In the past 20 years, researchers have become more and more interested in the electrical, mechanical, and physiological properties of muscle contraction, and they have started to incorporate electrophysiological findings into ENMS approaches. Working on the lower leg of the cat, Petrofsky described a computer-controlled stimulator and a special electrode array that could control the recruitment pattern of motor units during electrical stimulation (2). More recently, a sophisticated neuromuscular stimulation system was described that can be used to change forces in skeletal muscle by varying firing rates and recruitment control strategies (24). Using this system, Solomonow et al. systematically studied the power spectrum characteristics of the M-wave and the relation between force and EMG (5), and Baratta et al. (25) examined carefully the dependence of frequency response of muscle on control strategy.

To our knowledge, however, most early experimental designs using ENMS did not take into account the random nature of physiological motor neuron firing. Therefore, a potential gap exists between the research using periodic ENMS, and the findings obtained in electrophysiological studies of muscle. Physiological activation of motor units in intact skeletal muscle occurs through distributed random excitation. Distributed stimulation, in this context, means activating muscle according to the size principle, using multichannel stimulation. Random excitation refers to the random time interval between successive stimuli which may be described by a normal ISI distribution. Conventional electrical neuromuscular stimulation is typically performed using periodic or near periodic stimulation patterns. The coefficient of variation of the ISI of physiological and periodic activation of muscle is different: a coefficient of about 10 percent or higher for the physiological case (8,15,21), and a coefficient of 0 percent or close to 0 percent for periodic or near-periodic, artificial stimulations. It has been suggested that the magnitude of the coefficient of variation of the ISI affects the details of the EMG spectrum (8,10,15,16,21,26). In particular, changes in the median frequency of the EMG spectrum of periodically stimulated muscle are directly dependent on the mean stimulation rates because of the dependence of the spectrum on the coefficient of ISI variation. We hypothesize that the relation between the median frequency and the mean stimulation rate may not accurately reflect the actual relation between these parameters in the physiological case. The purpose of this study was to test analytically the relation between the median frequency of the EMG spectrum and the mean stimulation rate for varying coefficients of variation of the ISI, and for varying shapes of the compound motor unit action potentials (CMUAP, from MUAP, motor unit action potentials).

Based on findings from electrophysiological studies of skeletal muscle, we have used a distributed, random ENMS scheme that allowed for controlling the recruitment of motor units and the ISI statistics. The power spectrum of the EMG signal, commonly used in studies of voluntary muscular contractions (10,20,23,27,28), was chosen as an indicator of how well the EMG produced using the ENMS corresponded to that of isometric voluntary contractions. Since it is difficult to evaluate each particular ISI modification experimentally, a mathematical model, similar to that of myoelectric signal generation (8,10,17), was developed to predict the EMG spectrum of a muscle for a given stimulation design. The median frequency of the EMG power spectrum was calculated, and its dependence on the ISI statistics and the CMUAP was studied. Results of theoretical analyses confirmed experimental results reported previously using systematic, distributed, random stimulation of 8–10 ventral root filaments of cat soleus muscle (26,29).

METHOD

Advantages and Limitations of Periodic Stimulation Techniques

Periodic stimulation protocols are easy to implement, theoretically and experimentally, and they allow the study of many questions not directly concerned with simulating muscle behavior under physiologic, or near-physiologic, conditions. Periodic stimulation approaches are limited in elucidating the details of electrical, mechanical, and physiological properties of muscle during voluntary contraction, because EMG spectra (8,19), force production (29–33), rate of fatigue (unpublished observations), and many other neurophysiological features of muscle are different for periodic compared to non-periodic ENMS.

Strictly Periodic Stimulus Train

A strictly periodic stimulus train consists of a sequence of equally spaced, monophasic or biphasic, rectangular pulses. Since the duration of the stimulation pulses is much smaller than the duration of the MUAP, the stimulus train may be approximated by a sequence of impulses expressed by

$$x(t)=\sum_{j=-\infty}^{\infty} \delta(t-jT) \qquad [1]$$

where T is the reciprocal of the stimulus rate, λ :

$$\lambda=\frac{1}{T} \qquad [2]$$

The above approximation will not affect the conclusions drawn in the following analysis. The spectrum $\Phi_{xx}(f)$ of Equation 1 is obtained by

$$\Phi_{xx}(f)=\lambda^2\sum_{k=-\infty}^{\infty} \delta(f-k\lambda) \qquad [3]$$

λ which shows that the spectrum of the stimulus train is a set of delta functions with the areas of λ^2 at frequencies that are multiples of $\lambda = 1/T$, the stimulation rate. For stimulation rates within the physiological range of motor unit firing, conduction of action potentials is not inhibited; thus, the spectrum of the myoelectric response ($\Phi_{yy}(f)$) to the stimulation is

$$\Phi_{yy}(f)=\lambda^2\sum_{k=-\infty}^{\infty} \delta(f-k\lambda)\,|P(f)|^2 \qquad [4]$$

where $P(f)$ is the frequency response of the system for the generation of EMG signals in a channel (i.e., the Fourier transform of the MUAP, or CMUAP, if several motor units are stimulated through a single stimulation channel). According to Equation 4, the power density spectrum (PDS) of a periodic signal is a discrete function of frequency (frequency sampling, weighted by $|P(f)|^2$). A spectrum of this type is called a line spectrum, or discrete spectrum, and it does not adequately represent EMG spectra of voluntary contractions, which have been shown to be continuous functions of frequency (1,8,32,34). Similarly, for multichannel distributed stimulations, we have

$$\Phi_{yy}(f)=\sum_{i=1}^{m} \lambda_i^2\sum_{k=-\infty}^{\infty} \delta(f-k\lambda)\,|P_i(f)|^2 \qquad [5]$$

where m is the number of stimulation channels, λ_i is the stimulation rate of the ith channel, and $P_i(f)$ is the Fourier transform of the CMUAP of the ith channel. As shown above for the single channel scenario, the spectrum of the multichannel periodic stimulation will also be discrete. **Figure 1** shows the theoretical spectrum of an EMG signal obtained using four stimulation channels with different rates of stimulation: 26, 30, 45, and 50 pps. As a contrast to **Figure 1**, experimental spectra of EMG signals obtained from voluntary contractions of human rectus femoris muscle are shown in **Figure 2** (2); a similar result can also be found in (19). A similar spectrum can also be obtained during voluntary contraction in other muscles. It is evident that varying rates of stimulation in different channels does

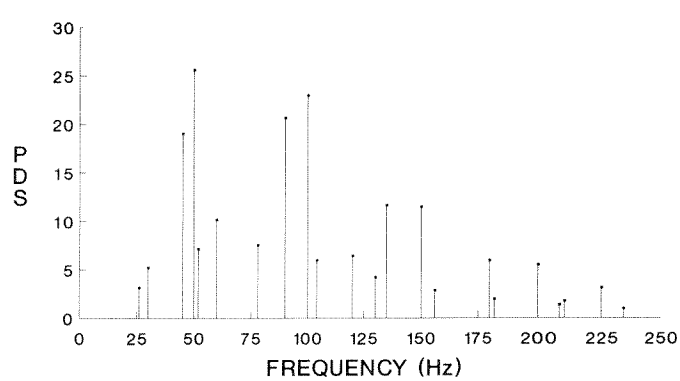

Figure 1.
Theoretical spectrum of the EMG signal obtained from four channels stimulated at periodic rates of 26, 30, 45, and 50 pps.

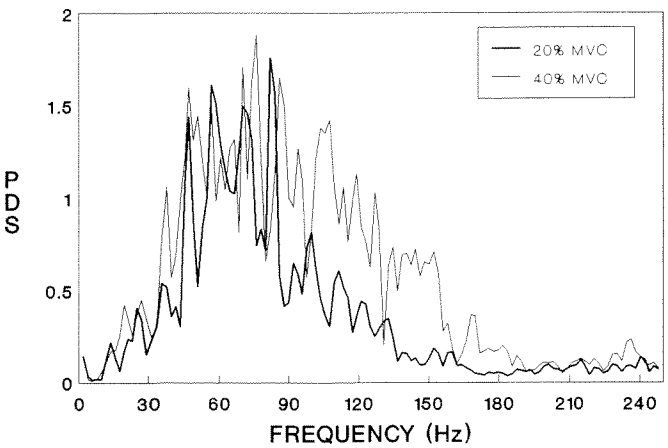

Figure 2.
Experimental spectra of EMG signals of voluntary contractions obtained from human rectus femoris at contraction levels of 20% and 40% of the maximum voluntary contraction (MVC).

not eliminate the discrete pattern of the spectrum if a periodic stimulation protocol is used. EMG spectral continuity cannot be achieved by recruiting additional motor units in a periodic ENMS protocol. However, recruitment may have an influence on the overall shape of the spectrum through the term $|P_i(f)|$ (Equation 5) which depends on the number of independent channels used for stimulation.

Practical Limitations of Periodic Stimulation

Periodic pulse trains are easy to implement, and they work well as long as physiologic properties of voluntary muscle contractions are not a major concern. However, if electrophysiological properties of muscle during voluntary contractions are the focus of study, there are at least three ways in which strictly periodic stimulation protocols are limited. First, periodic stimulations are limited in reproducing EMG spectra similar to those obtained during voluntary contractions. Stimulation-related information is carried in the temporal interval patterns of the CMUAP train. In voluntary contraction of muscle, these time interval patterns contain mainly low-frequency information (10,15,16,21,35). However, when using periodic stimulation protocols, the whole range of the EMG spectrum is affected, and thus periodic ENMS may obscure important details in the low-frequency regions of the spectrum.

Second, due to overlapping of muscle twitches and early depression effects, forces produced using periodic stimulation may differ from forces obtained using random ENMS (30,31), even for identical mean stimulation rates and recruitment strategies. As a consequence, the EMG-force relationship determined using periodic ENMS may not reflect the actual relationship that exists between these two parameters during voluntary contraction (29,32,33).

Third, the EMG spectra obtained using periodic stimulation become "discrete" (or line spectra) as shown in Equation 5. The use of this type of spectrum for determining the frequency response of muscle, or for studying systems identification, may have serious consequences, especially for high stimulation rates where only a few frequency samples are available in the EMG spectrum. This problem is enhanced when periodic stimulation is used to study vibromyographic signals or muscle sounds, which contain lower frequency components than the corresponding EMG signals (23,36–38).

Theoretical Basis For Distributed Random ENMS

The idea of using distributed random ENMS is based on findings from electrophysiological studies of skeletal muscle, specifically from a structural rather than a phenomenologic model of EMG signal generation. The structural model provides insight into how physiological parameters may contribute to observed EMG signals. It has been widely used in the areas of estimation and detection of myoelectric control (27), electrophysiological modeling (10), muscle tremor (35), performance analysis of myoelectric control channels (39–41), and generation and analysis of myoelectric signals (10,14,18). In this work, the structural model will be used to study systematically the effect of interactions between ISI patterns and CMUAPs on EMG spectra and their frequency parameters produced by distributed random ENMS. In particular, the relation between mean stimulation rate and median frequency of the EMG spectrum will be investigated as a function of varying coefficients of variation of the ISI statistics.

Spectral Expression for Distributed Multichannel Random Stimulation

A structural model for distributed ENMS is shown in **Figure 3**. In this model, $u_i(t)$, i = ,2,...,m, is the activation signal (stimulus train) which corresponds to the innervation signal for the fibers of the *i*th channel, and it is considered, as in the physiological case (10,27), a renewal point process with known activation statistics. Let $x_i(t)$, i = 1,2,...,m, represent the *i*th channel signal, then the autocorrelation function $\phi_{yy}(\tau)$ of the EMG signal y(t) can be written as

$$\Phi_{yy}(\tau) = E\left\{\sum_{i=1}^{m} x_i(t) \sum_{j=1}^{m} x_j(t+\tau)\right\} \qquad [6]$$

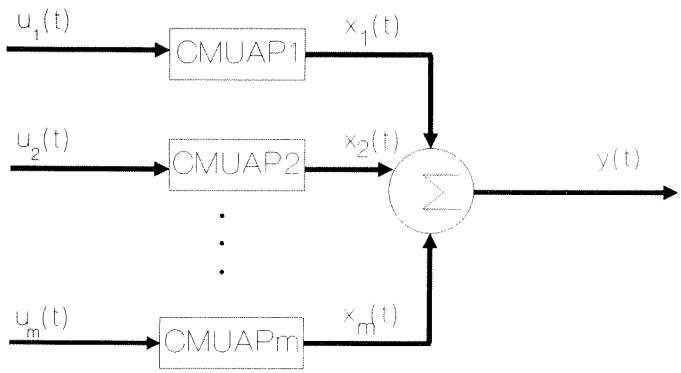

Figure 3.
Structural model of distributed ENMS with m stimulation channels.

where τ is the correlation lag value. With uncorrelated channels, the PDS, $\Phi_{yy}(f)$, is given by

$$\Phi_{yy} = (f) = \sum_{i=1}^{m} \Phi_{uui}(f) |P_i(f)|^2 \qquad [7]$$

where $\Phi_{uui}(f)$ is the PDS of the innervation process $u_i(t)$. The PDS of a single channel EMG signal consists of two parts: one part, $|P(f)|^2$, comes from the CMUAP owing to the occurrence of muscle activation, and the other part, $\Phi_{uui}(f)$, comes from the stimulation patterns of the *i*th channel. For a given channel, the PDS of the point process is found by (34,42),

$$\Phi_{uui}(f) = \lambda_i \left(1 + \frac{F_{ix}(f)}{1 - F_{ix}(f)} + \frac{F_{ix}^*(f)}{1 - F_{ix}^*(f)} \right), \; f \neq 0 \qquad [8]$$

where the superscript * represents the complex conjugate, and $F_{ix}(f)$ is the Fourier transform of the ISI probability density function $f_{ix}(x)$. Substituting Equation 8 for $\Phi_{ii}(f)$, Equation 7 yields

$$\Phi_{yy}(f) = \sum_{i=1}^{m} \lambda_i \left(1 + \frac{F_{ix}(f)}{1 - F_{ix}(f)} + \frac{F_{ix}^*(f)}{1 - F_{ix}^*(f)} \right) |P_i(f)|^2, \; f \neq 0 \qquad [9]$$

Equation 9 shows that 1) the EMG spectrum of simultaneous multichannel stimulation is the linear summation of the spectra for stimulation of the individual channels; and that 2) the spectrum of EMG signals does not only depend on the spectrum $P_i(f)$ of the CMUAP, but also on related physiological parameters: λ_i, the mean stimulation rate of each motor unit; m, the number of active motor units; and the pdf $f_{ix}(x)$ of the ISI.

Thus, any attempt of using the spectrum of an individual CMUAP as a measure of the entire EMG signal may, and typically will, give incomplete and misleading results. This problem occurs because the spectrum of a single CMUAP is, in general, not representative of the temporal and spatial summation of CMUAPS as a whole (Equation 9). Equation 9 allows for investigating the effect of interactions between the ISI statistics and the CMUAPs on the corresponding EMG spectra.

Spectral Properties of Random Gaussian Stimulations

Interspike intervals of voluntary contractions of skeletal muscles may be approximated using a Gaussian distribution (9). In this section, selected properties of the spectra produced by Gaussian stimulation protocols are summarized.

With $f_{ix}(x)$ representing a Gaussian distribution, Equation 9 may be written as,

$$\Phi_{yy}(f) = \sum_{i=1}^{m} \lambda_i \frac{\sinh[2(\pi f \sigma_{ix})^2]}{\cosh[2(\pi f \sigma_{ix})^2] - \cos(2\pi f/\lambda_i)} \; f \neq 0 \qquad [10]$$

where σ_{ix} is the standard deviation (SD) of the ISI for the *i*th channel. **Figures 4** and **5** show results obtained using Equation 10, for m = 1 and with different stimulation rates and coefficients of variation ($c_i = \sigma_{ix}\lambda_i$) of the ISI. In order to compare the spectral shifts for different values of the mean stimulation rate, for different values of the SD of the ISI, and for different coefficients of the ISI variation, the amplitudes of the PDS in **Figures 4** and **5** were normalized with respect to their maxima. Inspection of these figures and the corresponding equations indicates the following features:

1. The PDS, $\Phi_{uu}(f)$, for Gaussian point processes, has a high-pass characteristic. The band-pass width is controlled by the stimulation rate, λ, and the coefficient of variation of the ISI, *c*. This can be seen by the shift of the EMG spectrum as a function of the stimulation rate (**Figure 5**). At higher frequencies, the PDS, $\Phi_{uu}(f)$, is almost equal to the mean stimulation rate, λ (the normalized spectrum approaches 1 as the frequency increases). This result may be derived from Equation 10, for $f \rightarrow \infty$,

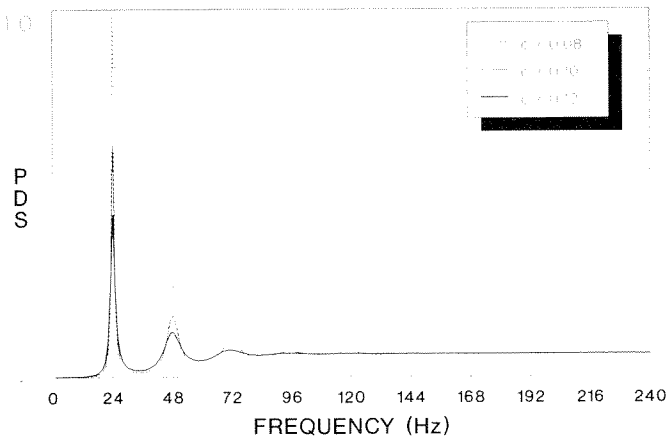

Figure 4.
Power density spectra of a Gaussian point process with $\lambda = 24pps$ and different values of c, where $c = \sigma_x\lambda$.

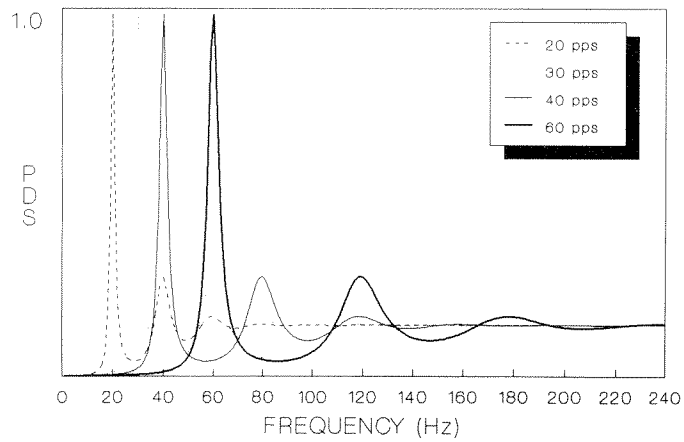

Figure 5.
Power density spectra of a Gaussian point process with $c = 0.1$ and different values of λ.

$$\lim_{f\to\infty} \Phi_{uu}(f) = \sum_{i=1}^{m} \lambda_i = \lambda_p, for\ \sigma_{ix} = 0 \qquad [11]$$

where λ_p is a pooled stimulation rate.

2. The PDS, $\Phi uu(f)$, has peaks at harmonics of the firing rate, depending on the form of the ISI pdf $f_x(x)$. This result may be inferred from Equation 10. With $f = k\lambda_i, k = 1,2,...$, the cosine factor in Equation 10 produces the peaks in the PDS.

3. The magnitudes of the peaks of the PDS that are caused by the mean rate of stimulation depend on the coefficient of variation of the ISI. This may be illustrated by substituting $f = k\lambda_i$ into Equation 10 as follows:

$$\Phi_{uu}(k,c) = \sum_{i=1}^{m} \lambda_i \frac{\sinh[2(\pi\, kc_i)^2]}{\cosh[2(\pi\, kc_i)^2] - 1}, f \neq 0 \qquad [12]$$

When c_i is small, the peaks of the PDS become large. The number of distinct peaks depends on the value of c_i. This statement may be verified using Equation 12, and it is illustrated in **Figure 4**. In the physiologic case, peaks of the PDS are pronounced in a frequency range from 0–120 Hz, depending on the value of c_i, and the shape of the CMUAP. This observation implies that the effect of ISI statistics on the power density spectrum of the CMUAP is mainly a low-frequency effect as reported in the literature (15,21).

4. The local minima in the PDS are given, at $f = k\lambda_i/2$ (k = 3,5,7,...), by

$$\Phi_{uu}(k,c) = \sum_{i=1}^{m} \lambda_i \frac{\sinh[(\pi\, kc_i)^2/2]}{\cosh[(\pi\, kc_i)^2/2] + 1}, f \neq 0 \qquad [13]$$

The magnitudes of these minima will change as a function of c_i and k as speculated by Christakos (35).

5. The PDS becomes discrete (line spectrum) as the coefficient of variation of the ISI approaches zero, $\sigma_{ix} \to 0$. Taking the limit $\sigma_{ix} \to 0$, Equation 10 gives

$$\lim_{\sigma_{ix}\to 0} \Phi_{uu}(f) = \begin{cases} \infty, f = \pm k\lambda_i, k = 1, 2,... \\ 0, \text{ other values of } f \neq \pm k\lambda_i, k = 0, 1, 2,... \end{cases} \qquad [14]$$

When $\sigma_{ix} = 0$, (i = 1,2,...m) or $c_i = 0$, the signal becomes periodic, and a periodic signal always has a line spectrum as shown above.

6. With $\sigma_{ix} = \sigma$ and $\lambda_{ix} = \lambda$, Equation 10 becomes

$$\Phi_{uu}(f) = \lambda_p \frac{\sinh[2(\pi f \sigma)^2]}{\cosh[2(\pi f \sigma)^2] - 1}, f \neq 0 \qquad [15]$$

This result shows that the spectrum of a stimulation protocol involving m channels with the same mean stimulation rate λ (but not necessarily identical stimulation patterns), and the same SD σ, will be equal to m times the spectrum of a stimulation protocol involving a single channel with λ and σ.

A Model for CMUAP

If several motor units are stimulated simultaneously in a ventral root filament, the resultant CMUAP, p(t), is the sum of the action potentials of the individual motor units in the filament. The CMUAP of a given filament will be invariant in form because of the fixed number of motor units activated, and the 'all-or-none' nature of action potential generation in motor units. The p(t) will not be identical to the muscle fiber action potential owing to diameter, endplate, and threshold dispersion, and it may not be identical across filaments for the same reasons, and because of the variation in the number of motor units in an activated filament. However, in general, the CMUAP may be expressed by summing the MUAPs activated in a single channel. One expression of a CMUAP that agrees well with observed data, and for which the spectrum can be derived analytically, is as follows:

$$p(t) = \sum_{n=1}^{L} h_n(t - \tau_n) \qquad [16]$$

where τ_n is a time shift ($\tau_1 = 0$), L is the number of motor units in a single channel, and h_n is the nth motor unit action potential in the filament which can be modelled by (27)

$$h_n(t) = \begin{cases} t(2 - b_n t)\exp(-b_n t), & 0 \leq t \\ 0, & otherwise. \end{cases} \qquad [17]$$

In this Equation, b_n is a constant, or a shape factor, which is determined by the size of motor units and the distribution of fiber types in a muscle. It can be shown that the spectrum of this CMUAP is characterized by the bandwidth control parameter b_n, and the time shift τ_n:

$$P(f) = \sum_{n=1}^{L} H_n(f)\exp(-j2\pi f \tau_n) \qquad [18]$$

where $H_n(f)$ is the Fourier transform of the MUAP $h_{n(t)}$ and is given by

$$H_n = \frac{j2\pi f}{(j2\pi f - b_n)^3} \qquad [19]$$

By properly selecting the parameters b_n, and n = 1,2,...L, a spectrum similar to that of a real CMUAP can be derived. In the ideal case, one stimulation channel should only include a single motor unit. In such a situation, $\tau_1 = 0$, $P(f) = H(f)$ and the amplitude of the spectrum may be obtained using

$$|P(f)|^2 = \frac{(2\pi f)^2}{[(2\pi f)^2 + b^2]^3} \qquad [20]$$

Spectra from Equation 20 with different values of the parameter b are shown in **Figure 6**. The spectrum shifts toward higher frequencies with increasing values of b.

Effects Of Interaction Between CMUAP and ISI Statistics On Frequency Parameters

Effects on the EMG Spectrum

After having examined the properties of the ISI statistics and the model for the CMUAP, it is possible to study the effects of interactions between the ISI statistics and the CMUAP on the PDS of the EMG signal. Equation 21 shows the EMG spectrum of an ENMS with a Gaussian ISI.

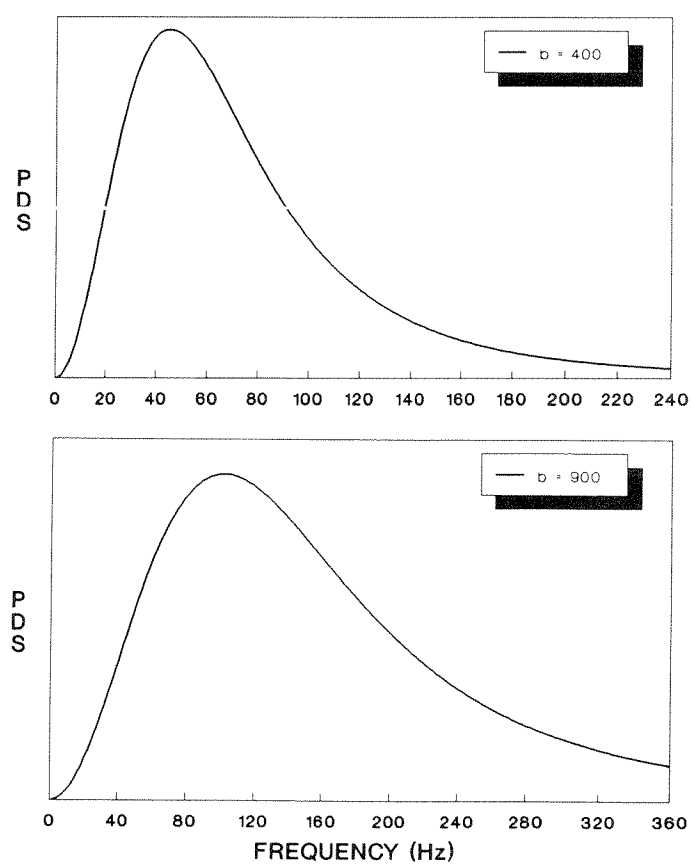

Figure 6.
Power density spectra of a MUAP with values of the parameter of $b = 400$ and $b = 900$.

$$\Phi_{yy}(f) =$$

$$\sum_{i=1}^{m} \frac{\lambda_i (2\pi f)^2 \sinh[2(\pi f \sigma_{xi})^2]}{\{\cosh[2(\pi f \sigma_{xi})^2] - \cos(2\pi f/\lambda_i)\}\{(2\pi f)^2 + b_i^2\}^3}$$

$$f \neq 0 \qquad [21]$$

The EMG spectrum in Equation 21 is plotted in **Figures 7a, 7b,** and **7c** for a CMUAP with $b = 900$ and in **Figure 8** for a CMUAP with $b = 400$, for different values of λ and σ_x. These figures illustrate the changes in the EMG spectrum associated with the ISI statistics under various conditions. The influence of the ISI statistics on the PDS is mainly concentrated around the low-frequency region when σ is small (**Figures 7a** and **8**). Peaks are introduced at multiples of the firing rate, as observed experimentally in electrophysiological studies (16,21). Such changes in the PDS as a function of the ISI statistics may reflect variations in activation. For example, when activation levels increase, it is expected that the firing rate will increase, and as a result, the peak of the spectrum will shift toward higher frequencies. It is also noted from these figures, and the corresponding equations, that the magnitude of the peaks of the PDS depends on the coefficient of the ISI variation.

With a Gaussian ISI, $\sigma_{ix} = \sigma$ and $\lambda_{ix} = \lambda$, Equation 9 becomes

$$\Phi_{yy}(f) = \lambda \frac{\sinh[2(\pi f \sigma)^2]}{\cosh[2(\pi f \sigma)^2] - 1} \sum_{i=1}^{m} |P_i(f)|^2, \; f \neq 0 \; [22]$$

Equation 22 gives the PDS of a distributed, random ENMS, where stimulations in each channel have similar rather than identical statistical properties. When the CMUAPs are approximately the same across channels, the PDS of the EMG signal has the same form as that of an individual channel, except that the multichannel PDS is scaled by a factor m, where m is the number of channels used in the ENMS. This result represents the so called grouping effect of action potentials and has been reported in electrophysiological studies (35).

Effects on the Median Frequency

In order to quantify changes in the myoelectric spectrum as a function of the firing rate and SD of the interspike interval, the median frequency, f_{med}, of the signal was calculated numerically using Equation 21 (with m = 1) as a function of λ. The plots of the median frequency against λ are shown in **Figure 9** for different values of σ_x and in **Figure 10** for different values of the parameter b.

The relationship between the median frequency of the EMG spectrum and λ is nonlinear, especially for small σ_x. This result is associated with the nonlinear relation between $\Phi_{yy}(f)$ and λ, as indicated in Equation 21, and in

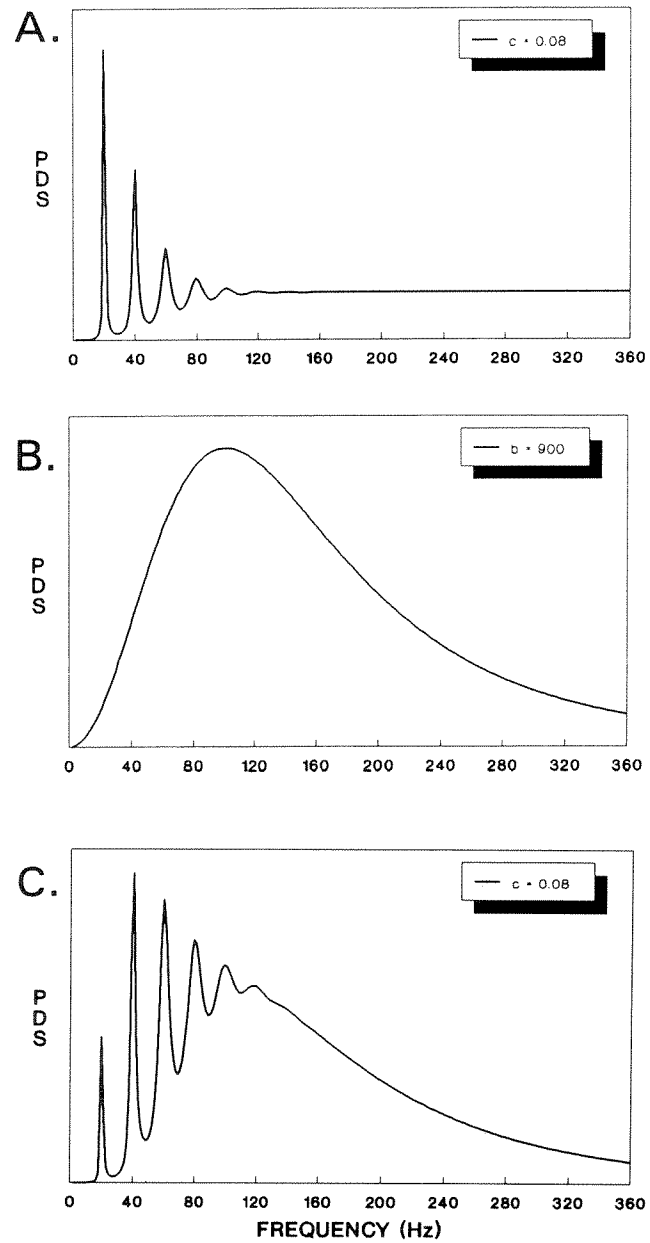

Figure 7.
Influence of the ISI statistics on the PDS of the EMG signal: (a) PDS of a Gaussian point process with $\lambda = 20pps$; (b) MUAP with parameter $b = 900$; and, (c) PDS of the EMG signal.

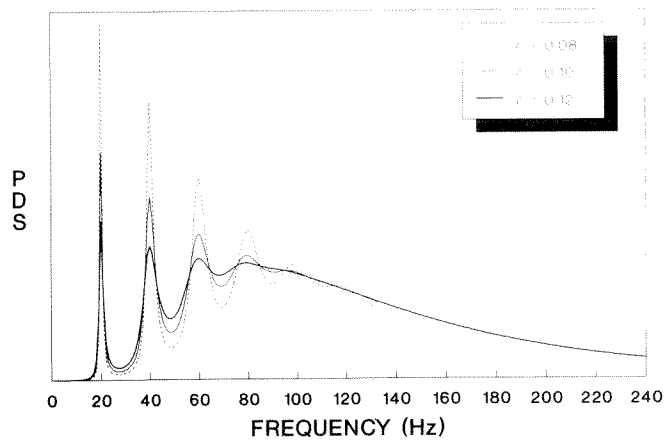

Figure 8.
Effects of changes in the coefficient of variation of the ISI on the PDS of the EMG signal with $\lambda = 20pps$.

Figures 8 and **9**. For large values of σ_x, the change in the median frequency as a function of λ becomes small (e.g., **Figure 9**, $\sigma_x = 8$ms), because the fact that the effect of changing λ on the EMG spectrum decreases with increasing σ_x. Nevertheless, in all cases, the median frequency tends to increase with increasing λ. These data support experimental results reported in the literature (21).

The observation that changes in the median frequency, as a function of λ, become small for large values of the σ_x is further demonstrated theoretically in the Appendix using m = 1. It was found that for large values of the SD of the ISI, the median frequency relates to parameter b of the MUAP as,

$$f_{med} \leq 5|b| \qquad [23]$$

Equation 23 indicates that the median frequency is independent of the rate of stimulation (or firing), when the values of the σ_x are large, but depends on the CMUAP through the parameter b. The larger the parameter b, the higher the median frequency. In general, the median frequency depends on both stimulation (or firing) statistics, and the MUAP, and thus is an indicator of the combined effects of nerve stimulation and muscle response to the nerve stimulation.

Experiments on Cat Soleus Muscle

Experimental data were obtained from the cat soleus muscle. Cats were anesthetized and placed in a stereotaxic frame (43) in a prone position with the hind limbs rigidly fixed. Ventral roots L7 and S1 were exposed, separated, and carefully divided into bundles (29). Each of these bundles was hung over a separate bipolar electrode for individual and simultaneous distributed stimulations. The stimulations, using patterns generated from computer simulations, were applied to the ventral root bundles via the electrode. EMG signals from stimulations of each of the 10 nerve bundles individually (finger prints), and the simultaneous stimulation of all 10 bundles were measured using a pair of indwelling bipolar electrodes. EMG signals were digitized on line and stored on a computer. Blood

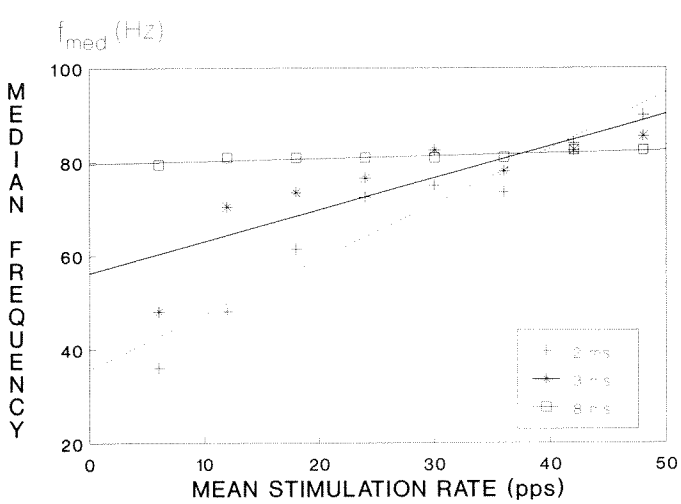

Figure 9.
Median frequency of PDS versus λ for $\sigma_x = 2$, 3 and 8 ms. (Symbols: the calculated median frequency, Lines: the regression line approximation).

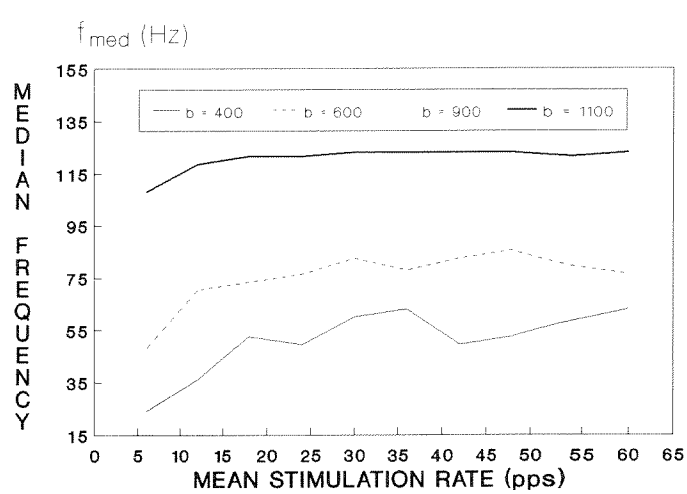

Figure 10.
Median frequency of the PDS as a function of λ for different values of the MUAP parameter b.

pressure, core temperature, and muscle temperature of the cats were monitored continuously and were kept constant throughout the experiment.

The power density spectrum of each experimental EMG signal was estimated using a 512-point fast Fourier transform (FFT) algorithm, and then averaged over four consecutive segments in order to reduce estimation errors. **Figure 11** shows two representative spectra of experimental EMG signals obtained using the same value for μ and different values for c. The experimental results support the mathematical predictions discussed in the previous sections. Specifically, the EMG signal spectrum shows peaks at the mean stimulus rate and its multiples when the coefficient of the ISI variation is small; and the envelope of the PDS is determined by the CMUAP waveform and the details of the ISI stimulus statistics.

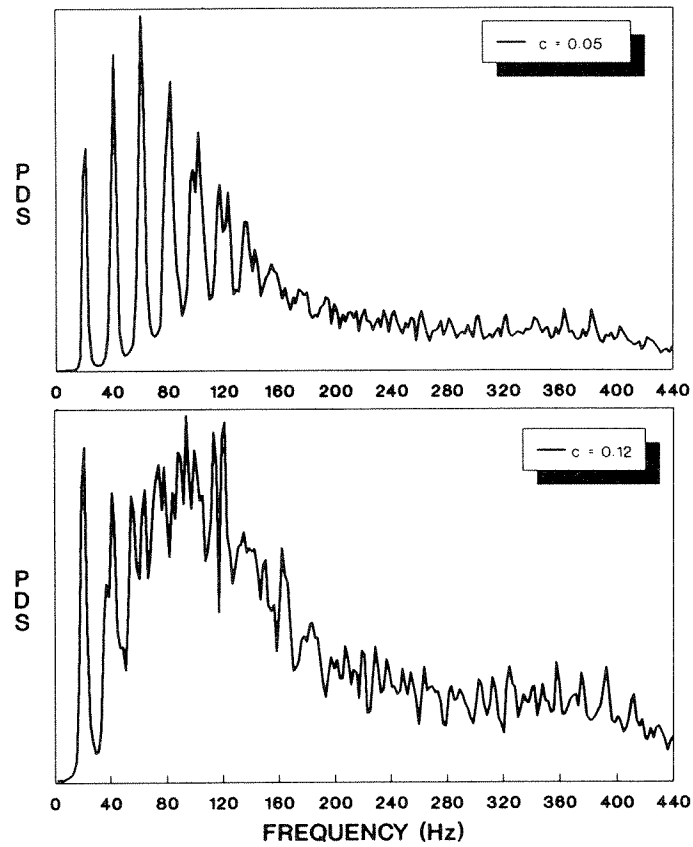

Figure 11.
PDS of EMG signals obtained from random ENMS with $\lambda = 20pps$ and different values of c.

RESULTS AND DISCUSSION

General Discussion

Figure 8 shows the effects of changes in the SD of the ISI on the single channel EMG spectrum. A decrease in the SD of the ISI produced an increasingly more discrete spectrum of the EMG, indicating that variations of the ISI play an important role in controlling the details of the corresponding EMG spectrum. This finding was supported by our experimental results (**Figure 11**). A comparison of the experimental results of the ENMS (**Figure 11**) with the spectra obtained during voluntary contraction in the human rectus femoris (**Figure 2**) indicates that the EMG spectrum obtained using ENMS with a coefficient of variation of the ISI of 12 percent is similar to that obtained for voluntary contractions. Therefore, in situations where it is necessary to produce EMG spectra and frequency parameters similar to those obtained during voluntary contractions, it is suggested that variations in the inter-stimulus intervals be introduced.

The theoretical results of this study indicate 1) that the EMG spectrum shows peaks at the mean stimulation rate and its multiples; 2) that the magnitude, or clarity, of the peaks at the mean stimulation rate and its multiples depends on the coefficient of variation of the ISI, and further, that the PDS approaches a line spectrum when the coefficient of variation of the ISI becomes small (**Figure 8**); and, 3) that the envelope of the PDS is primarily determined by the shape of the CMUAP, and is virtually independent of the ISI statistics when the coefficient of variation of the ISI is large. This result is illustrated by the similarity of the envelopes of the spectra in **Figures 7** and **8**. It has been found that, from a mathematical point of view, there is a distinct difference in EMG spectra between periodic and random ENMS. It is possible to obtain "normal" EMG spectra (as defined here) with random ENMS, but not with periodic ENMS.

The median frequency of the PDS depends on the interaction of the firing (or stimulation) statistics and the MUAP. The results reported here imply that it is possible to use the median frequency as a selective indicator of changes in the MUAP by increasing the SD of the inter-stimulus interval in distributed random ENMS to sufficiently large values (8 ms or higher, see **Figure 9**). This approach may be used when attempting to determine qualitatively the contribution of variations in CMUAP to a change of the median frequency using random ENMS.

In summary, three features distinguish the present study from previous ENMS investigations (6). First, emphasis was placed on introducing random ISI into the ENMS. Second, the distributed multichannel stimulation

approach allows the simulation of recruitment of motor units according to the size principle. This is an important feature of physiologic activation. Third, the effects of the mean stimulation rate on the EMG spectrum and the frequency parameters of stimulated muscles were studied analytically for varying coefficients of variation of the ISI and for varying shapes of the CMUAP. The theoretical findings presented in this study are supported by the experimental findings using a distributed, random ENMS approach.

Clinical Relevance

The median frequency of the EMG spectrum has been used widely to assess muscular fatigue. When muscles fatigue, the median frequency of the EMG signal tends to decrease (8). However, in experiments using voluntary muscular contraction, it is difficult to determine how much of the change in the median frequency is caused by changing in the local muscle properties (as reflected in the MUAP) and how much is caused by the decrease in firing rates of motor units. Distributed random ENMS provides a possibility for separating these two factors that influence the median frequency by choosing the SD of the ISI appropriately.

CONCLUSION

The distributed, random ENMS approach described here can be used to study the properties of EMG spectra in situations approximating voluntary contractions; however, simply reproducing EMG spectrum of voluntary contraction is not sufficient to guarantee mechanical muscle behavior similar to that of voluntary contractions. The effect of changes in stimulation parameters, such as the mean stimulation rate, the number of stimulation channels, and the coefficient of variation of the ISI, on muscle force, EMG, and VMG signals must be determined analytically and experimentally.

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APPENDIX

LIST OF SYMBOLS

A—the average amplitude of the MUAP
b—the shape factor of motor unit action potential
c—the coefficient of variation $c = \sigma_x\lambda$
CMUAP—the compound motor unit action potential
$E[\ ]$—the expectation operator
ENMS—electrical neuromuscular stimulation
$f_x(x)$—the probability density function of random inter-stimulus interval x
$F_x(f)$—the Fourier transformation of $f_x(x)$
$h(t)$—the CMUAP
ISI—the inter-stimulus interval (or the inter-spike interval)
m—the number of stimulation channels
MUAP—the motor unit action potential
pdf—probability density function
PDS—power density spectrum
$P(f)$—the Fourier transform of $p(t)$
pps—pulses per second
$p(t)$—the averaged MUAP
$r(t)$—the renewal point process
SD—standard deviation
T_i—the excitation time instant
T_o—the duration of the MUAP
$var[\ ]$—the variance operator
x_i—the ith ISI
λ—the mean stimulus (firing) rate
σ_x—the standard deviation of the ISI
$\phi(\tau)$—the autocorrelation function
$\Phi(f)$—the power density spectrum

The Relation between Median Frequency and the Parameter *b*

When σ is large, we have from Equation 22 the approximation

$$\Phi_{yy}(f) \cong \sum_{i=1}^{m} A_i\lambda_i \frac{(2\pi f)^2}{\left\{(2\pi f)^2 + b_i^2\right\}^3} \qquad [A1]$$

According to the definition of the median frequency, we get

$$\int_0^{f_{med}} \sum_{i=1}^{m} A_i\lambda_i \frac{(2\pi f)^2}{\left\{(2\pi f)^2 + b_i^2\right\}^3} df = \int_{f_{med}}^{\infty} \sum_{i=1}^{m} A_i\lambda_i \frac{(2\pi f)^2}{\left\{(2\pi f)^2 + b_i^2\right\}^3} df \qquad [A2]$$

For m = 1, the above equation reduces to

$$\int_0^{f_{med}} \frac{(2\pi f)^2}{\left\{(2\pi f)^2 + b^2\right\}^3} df = \int_{f_{med}}^{\infty} \frac{(2\pi f)^2}{\left\{(2\pi f)^2 + b^2\right\}^3} df \qquad [A3]$$

Solving the definite integrals on both sides (with $x = 2\pi f_{med}$ and $e = b^2$), we get

$$\frac{x}{\pi\left\{(x/2\pi)^2 + e\right\}^2} + \frac{x}{2\pi e\left\{(x/2\pi)^2 + e\right\}} + \frac{1}{e^{3/2}}\, arctg\left(\frac{x}{2\pi e^{1/2}}\right) - \frac{\pi}{4e^{3/2}} = 0 \qquad [A4]$$

For $x < 2\pi e^{1/2}$, we have

$$artg\left(\frac{x}{2\pi e^{1/2}}\right) = \frac{x}{2\pi e^{1/2}} - \frac{x^3}{3(2\pi e^{1/2})^3} + \ldots\ldots \qquad [A5]$$

Using the first order approximation in Equation (A4) and considering $e = b^2$, we obtain

$$\frac{x^5}{2(2\pi b)^4} - \frac{x^4}{4b^3 2^4 \pi^2} + \frac{3x^3}{2b^2(2\pi)^2} - \frac{x^2}{8b} + 2x - \pi^2 b = 0 \qquad [A6]$$

Substituting $f_{med} = x/2\pi$ into the above Equation (A6), we finally get

$$\left(\frac{f_{med}}{b}\right)^5 - \frac{\pi}{\pi}\left(\frac{f_{med}}{b}\right)^4 + 3\left(\frac{f_{med}}{b}\right)^3 - \frac{\pi}{2}\left(\frac{f_{med}}{b}\right)^2 + 4\left(\frac{f_{med}}{b}\right) - \pi = 0 \qquad [A7]$$

The upper bound of the root Υ for the general equation

$$a_0 x^n + a_1 x^{n-1} + \ldots + a_{n-1} x + a_n = 0 \qquad [A8]$$

is given by

$$|\Upsilon| \le 1 + \frac{1}{a_0} \max\left\{|a_1|, |a_2|, \ldots, |a_n|\right\} \qquad [A9]$$

and

$$f_{med} \le 5\,|b| \qquad [A10]$$

A more specific root of Equation A10 could be obtained using the Newton Integration approach. For our purpose, Equations A7 and A10 are sufficient to indicate that the median frequency is independent of the stimulation (or firing rate) for large values of the standard deviation of ISI but depends on the CMUAP, which supports the observations made in Figure 6 and Equation 21.