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CLINICAL REPORT

Application of Fuzzy Logic Techniques for the Qualitative Interpretation of Preferences in a Collective Questionnaire for Users of Wheelchairs

Rubén Lafuente, MS; Álvaro Page, PhD; Javier Sánchez-Lacuesta, MS; Lourdes Tortosa, MD
Institute of Biomechanics of Valencia (IBV) 46980-Paterna, Valencia, Spain

Abstract—Active participation of users in the evaluation of technical aids is essential, since they are part of the interface with the system and constitute a fundamental source of design criteria. In this study, 88 active users of wheelchairs were interviewed by means of a written questionnaire about their opinion concerning the adaptation of his/her wheelchair to the office workplace. A conceptual framework was introduced linking objective measurements of the user-wheelchair interface to the subjective preferences expressed by the user. Discriminant analysis was used in order to select and quantify the importance of the most significant factors influencing the user's opinions. Fuzzy logic was introduced for the qualitative interpretation of the relationship between those significant factors, based on an inductive algorithm for generating fuzzy rules. Fuzzy logic enables a person to model the uncertainty within the subjective formulation of knowledge or opinions. From the results, a mismatch between actual performance of conventional wheelchairs and requirements of office work became evident. The proposed methods make it possible to determine reliable rules explaining subjective preferences; thus, they provide a flexible means of interpreting user questionnaires and obtaining new design criteria.

Key words: *fuzzy logic, qualitative models, questionnaires, wheelchair design.*

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Address all correspondence and requests for reprints to: Rubén Lafuente, MS, Institute of Biomechanics of Valencia (IBV), P.O. Box 199, 46980-Paterna, Valencia, Spain.

INTRODUCTION

The common interest in design enhancement of technical aids for persons with disabilities, aiming at improving product quality and user level of satisfaction, has led to the development of new evaluation techniques. As an illustration, considerable efforts have been devoted to the area of the technical assessment of wheelchairs. Technical standards have been developed since the mid-1960s due to the concern of consumers, manufacturers, and governments for the safety and availability of quality products (1), resulting in a set of wheelchair standards approved by several organizations (ISO, CEN, ANSI/RESNA, and so forth). Various projects have also been undertaken focusing on the ergonomics of wheelchair driving (2), as well as on the set-up of a methodology for consumer evaluation of wheelchairs (3).

Objective evaluation is primarily based on criteria established from domain experts and knowledge acquired from controlled experiences. Laboratory experiments are an important source of information, but they are costly due to the instrumentation needed, and are sometimes inaccurate because of the limited number of test subjects and the difficulty of simulating real conditions. Furthermore, assessment criteria cannot be optimized, or sometimes even discovered, without the participation of users encouraging the detection of design faults and the proposal of new solutions (4). The same idea appears in the conclusion of a recent study funded by the European Community addressing the development of technological tools and applications for people with disabilities and older people (the HEART project): "the need for user representatives to be involved in all instances where issues on disability are treated, in order to influence policies and programmes, such as creating methods for user feed-back on product development" (5).

Consequently, alternative approaches employing user surveys may be a proper means to tackle this problem. A typical measurement tool for subjective assessment in field analyses is a written or oral questionnaire, in which the preferences of the users, regarding several ergonomic and design topics of the technical aid, are studied. Current methods of interpretation are based mainly on descriptive statistics, such as tables of contingency, means, variances, and so forth. When the aim is to simultaneously explore several design variables, multivariate statistical methods (i.e., Logistic Regression or Discriminant Analysis) are the most appropriate.

The advantages of using statistics are the handling of quantitative data, the good empirical and statistical foundation of the methods and their powerful capability of generalization. The major drawbacks are the need to check relevant *a priori* conditions in order to be applied (normality, statistical independence, and so forth), and the expertise required to process results. Moreover, statistics are based on a clear definition of the objects, factors, and categories involved in the study, which must be specified without ambiguity. However, when dealing with preferences and opinions, the use of accurate linguistic terms is neither adequate to formulate questions to users, nor the most suitable way of processing their answers. An alternative approach offering a friendlier interface for the formulation of "fuzzy" concepts could be of benefit for the identification of user preference in assistive technology.

Fuzzy logic was first introduced by Zadeh (6) as a result of the logical paradoxes detected in common engineering applications. Fuzziness measures the extent to which an event may occur or to which an entity may be classified as something. Fuzzy logic permits variables to belong to more than one set or class, thus enabling computers to cope with vague concepts (e.g., the temperature of a room might be appropriate, but a bit high at the same time). In this way, the development of a complex mathematical model of any technical system can be replaced by a simpler qualitative representation.

The fundamental feature of fuzzy logic is its capability to model the membership of an element to a fuzzy set by means of a continuous function. Whereas the membership is either complete or absent in traditional terms, this relationship can continuously change in fuzzy logic from inexistent to full membership. Fuzzy sets can be linguistic concepts difficult or even impossible to designate in exact terms (e.g., "big," "very small," "quite good," etc.), but also other sorts of variables that are fuzzy in nature (e.g., all variables describing a person's psychological aspects). A membership function $\mu_A(x)$ assigns a degree of certainty in the interval [0; 1] to the statement "Element x belongs to the fuzzy set A." In this way, fuzzy logic simplifies the formulation of knowledge, since it enables the generation of rules including fuzzy variables, so that the expert is less constrained in expressing and interpreting knowledge. **Figure 1** shows the association of three linguistic variables (fuzzy sets) to a continuous dimensional variable.

Most applications of fuzzy systems are intended for the areas of process control and data analysis (7). Particularly in the medical domain, the use of fuzzy logic has made knowledge acquisition and manipulation easier, enabling the development of new expert systems that are more robust in their way of inferencing responses and more human-like in the process of obtaining and producing knowledge (8–10). Following

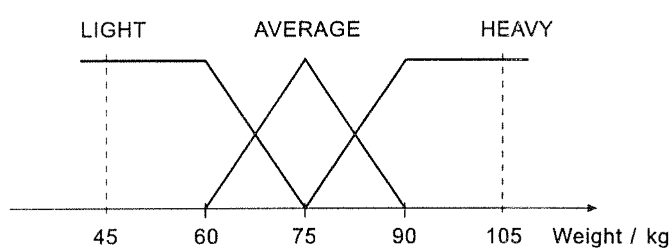

Figure 1.
Three membership functions associating three linguistic terms ("light," "average," and "heavy") to the continuous and measurable magnitude "weight." There are regions in the weight axis where the membership to a fuzzy set is multiple, which means that one specific weight value can be, for instance, medium and heavy to different degrees at the same time.

this approach, fuzzy expert systems could also be implemented for the qualitative assessment of technical aids, since information for their design can be mainly provided either by experts or by users.

One particular problem of a fuzzy system is the way of determining its rules. Most systems are designed from the experience of domain specialists or from the specific knowledge found in the literature. Only when this special knowledge becomes available, can the approach lead to a useful fuzzy system. Inductive systems, on the other hand, generate knowledge rules directly from the information gathered by means of measurements, providing assistance when specific knowledge is either elusive to obtain or too subjective. Inductive systems are designed with the help of a training algorithm, which tries to match optimally the model's predictions to the real observations fed into the system during training. In this context, neuro-fuzzy networks (11–13), adaptive clustering techniques (14–16), and genetic algorithms (17–19) have been proposed. A major drawback of inductive expert systems is their dependency on the training data set, which makes validation with other data necessary.

This article presents a new algorithm for the automatic generation of fuzzy rules from measured information either of physical origin (e.g., dimensions of the technical aid) or of subjective nature (e.g., user opinion). Here it is applied to the qualitative interpretation of preferences of users of wheelchairs concerning their occupational activity at office. A questionnaire carried out among 88 users working in different offices was used to obtain the data, and the results related to the functional and ergonomic factors that played a key role in global user satisfaction are presented to demonstrate the validity of the proposed methods.

METHODS

The first part of this section describes a questionnaire that was used to characterize the user-wheelchair interface and to obtain user feedback in a field study where data were obtained. The second part deals with the theoretical rationale for wheelchair evaluation that was used to create the fuzzy logic model and the methods applied to interpret and test that model. The Glossary, which follows the Appendix, provides definitions of the particular terminology employed throughout the text.

Field Study

Scope

The questionnaire that was used stemmed from a previous investigation carried out at the Institute of Biomechanics of Valencia (20) among active users of wheelchairs with the aim of determining and quantifying the most relevant problems in the user's office workplace, particularly regarding accessibility, furniture design, and task performance.

The questionnaire was filled in by trained staff while the user was working. Due to practical reasons, the measuring process was conducted at the office with the user seated in his/her wheelchair. That implied serious limitations and made the determination of some dimensions (e.g., seat angle) only indirectly possible. The procedure consisted of the following parts:

- a questionnaire of subjective user preferences
- measurement of anthropometric dimensions of the user while seated
- measurement of wheelchair dimensions
- measurement of other features of the office environment.

Since the questionnaire was intended for assessing the occupational conditions of users of wheelchairs, some measurements included in the original forms related to furniture design. From the previous study, it could be concluded that intervention on workplace design was not a priority goal in our country, but that the basic factor influencing user-satisfaction for office work was the design of the wheelchair itself. Intervention on wheelchair design seemed more sensible in this context, thus concentrating on variables that configure this design while not taking any other environmental factor into consideration for this study.

Questionnaire of Subjective Preferences

Table 1 displays the subjective preferences, asked of the user about his/her wheelchair, that were considered valuable for the purpose of the present study. First, the user was asked about his/her level of satisfaction; then his/her opinion regarding several functional aspects was investigated. For this, it was assumed that these functional aspects influenced the user's level of satisfaction. Furthermore, the dependence of these functional aspects on the dimensional variables of the wheelchair-user interface was analyzed.

User feedback was input in three linguistic levels, because it was easier for the subject to answer in this manner. The analysis of the user's preferences was

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Table 1.
Subjective preference questionnaire.

Preference	Evaluation levels
General satisfaction level regarding the wheelchair	Totally satisfied, Partially satisfied, Not satisfied at all
Capability of adjusting the chair according to the demands of office job	Good, Bad
Aesthetic outlook of wheelchair in user's opinion	Nice, Average, Ugly
Seating comfort for office work	Comfortable, Average, Uncomfortable
Ease of maneuvering the wheelchair	Easy, Average, Difficult
Ease of overcoming small obstacles	Easy, Average, Difficult
Price-quality ratio in user's opinion	Good, Average, Bad
Ease of pushing the wheelchair	Easy, Average, Difficult
Feeling of safety during wheelchair use	Safe, Average, Unsafe
Wheelchair durability considering frequency of repairs	Good, Average, Bad

carried out only in two levels (a positive and a negative opinion), because the sample size and sample variability did not allow for a finer categorization in the statistical analysis.

Measurements of Users and Wheelchair Dimensions

Eighty-eight subjects between the ages of 18 and 55 years (average 36 years), from all over the country, were analyzed in the study; 54 were men and 34 women. All used manual wheelchairs due to different pathologies of the users: paraplegia (41 percent), poliomyelitis (23 percent), tetraplegia (13 percent), muscle dystrophy (8 percent), cerebral palsy (5 percent), and other diseases (8 percent).

Most subjects (65 percent) showed normal mobility in the upper limbs and 75 percent had paralysis in the lower limbs. The functional condition of the subjects investigated was rather satisfactory. A functional questionnaire was administered to each subject regarding several activities of daily life, in which the majority (63 percent) reached the maximum score of 60 points in the scale.

The sort of occupational activities carried out by the subjects were always concomitant to the use of a desk and other pieces of office furniture and often also of a computer. Fifty-one percent of the subjects interviewed reported some kind of body discomfort referring its origin to their job rather than to their disabilities. The dimensions of the subjects were measured with a Martin anthropometer, but were not included in the modeling process.

The wheelchairs that were analyzed were all of manual propulsion and their use was general and not restricted to only the office. **Figure 2** depicts the wheelchair dimensions that were determined by means of a tape measure. Seat angle was calculated from geometrical considerations using the anterior (FW) and posterior seat height (GW) and seat depth (MW).

Since aspects related to the adaptation of the wheelchair to the office work were ranked by the users themselves as higher than other aspects related to mobility, some measurements that might also have been regarded for the analysis of propulsion, maneuverability, or safety, can be omitted in the present study.

Methodology for the Qualitative Modeling of User Preference

Framework for Interpreting User Preference

The process of ergonomic evaluation of the wheelchair was based on assuming the existence of a deterministic cause-effect chain between the physical design of the user-aid interface and the final consequences of its use on the subject (comfort/discomfort, facility/difficulty of use, and so forth). The modeling process was based on assuming a relationship between functional aspects of the wheelchair and objective dimensions of the user-aid interface, and a dependence

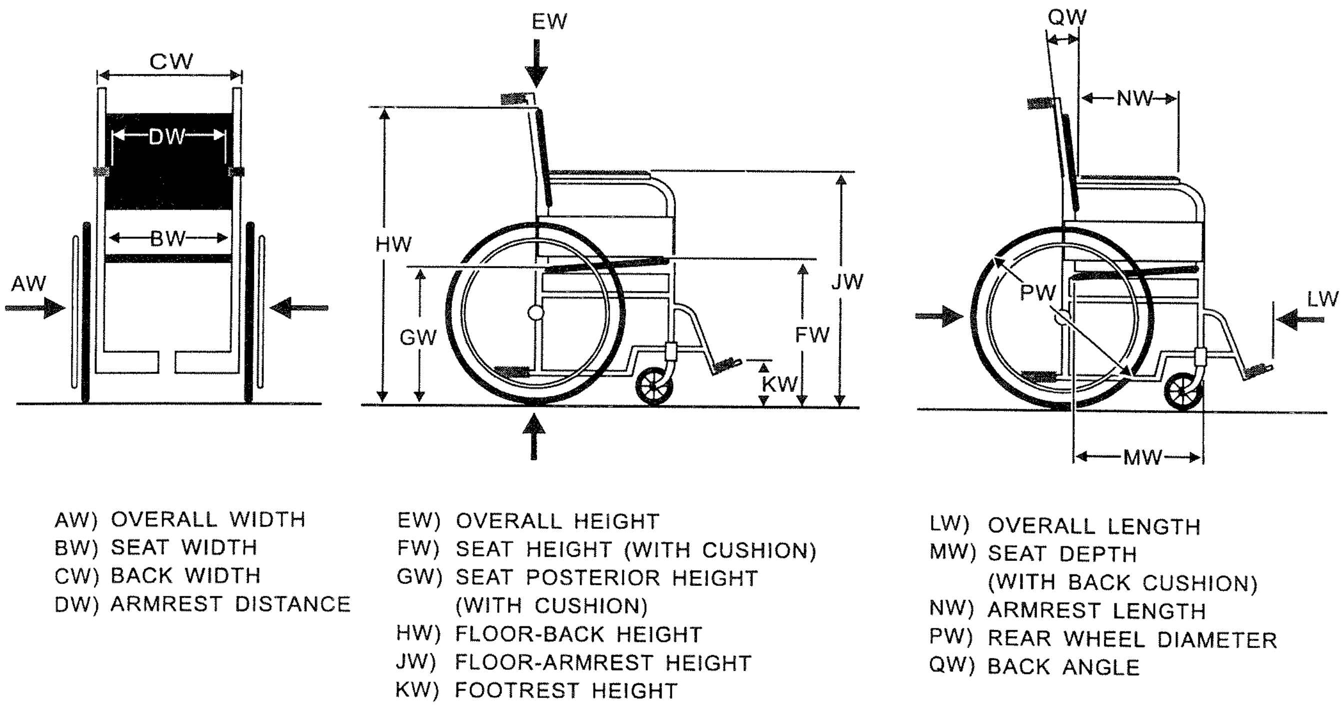

Figure 2.
Wheelchair dimensions measured in the questionnaire.

between those functional aspects and the global opinion of the user about his/her technical aid.

Figure 3 separates this concept into variables analyzed, techniques deployed, and results obtained. Significant functional aspects of the wheelchair that influenced the global satisfaction of the user were discovered and ranked in importance by means of a statistical technique described in the section headed *Selection of Significant Input Variables.* Internal relationships among those factors were modeled by means of the fuzzy algorithm explained in the section headed *Interpretation of the Relationship Between Input Variables: Fuzzy Logic.* The fuzzy model was then reduced to a set of rules, which also enabled the prediction of users' opinions. Using the same methods, significant objective measurements of the user-wheelchair interface were selected and their interrelation to explain the significant functional aspects of the wheelchair identified. In the former instance, qualitative variables (preferences) were used as model inputs, whereas inputs were of quantitative nature (dimensions) in the latter.

Different preferences may be expressed depending on specific user characteristics (i.e., body size, sex, age, physical condition) or environmental factors (e.g., furniture design, working conditions). Since the sample size was not too large, it was assumed that the preferences expressed by the interviewed subjects were independent of each user's size and functional status, and the environmental factors. The dimensions of the subjects were, therefore, not taken into account and only variables characterizing the user-wheelchair interface were considered as potentially significant factors (**Figure 2**). The error conveyed by this assumption did not preclude an appropriate interpretation of results.

Selection of Significant Input Variables. Fisher's Discriminant Analysis (FDA)

When subjects' answers are expressed in previously defined evaluation levels (e.g., easy, average, difficult), classification techniques can be employed in order to predict the likelihood of an input parameter vector to belong to each of these linguistic classes. Fisher's Discriminant Analysis (FDA) is a powerful

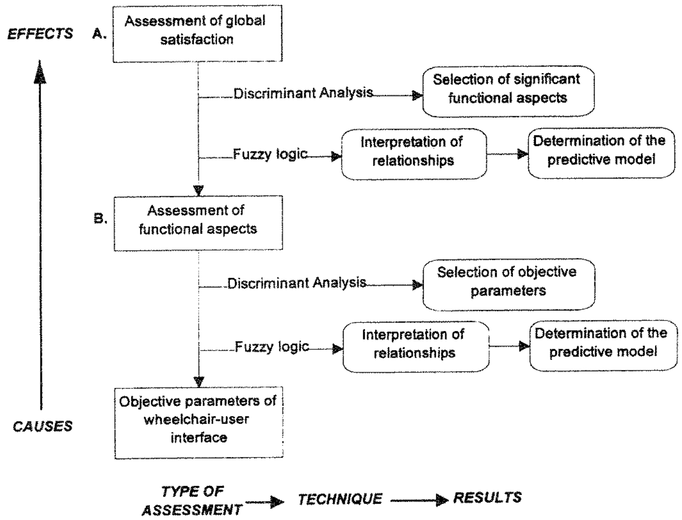

Figure 3.
Framework for modeling the preferences of user. A cause-effect chain between objective parameters defining the wheelchair-user interface and the final opinion of the user is assumed. Global satisfaction is assessed in the dependence of several functional factors; significant functional factors are then analyzed in dependence of objective dimensions of the wheelchair-user interface.

linear multivariate classification tool used to maximize the ratio of variability observed between different classes and the variability observed within the classes (21,22). The relative importance or discrimination potential of each input parameter can be estimated by means of the absolute value of the standardized discriminant coefficients (SDC) $\alpha_{j,i}$.

If K is the total number of classes defined and n the number of inputs, FDA calculates f=min $(K-1,n)$ projection axes z_j (j=1,..,f) of maximum separation between the groups. Each projection axis z_j represents a new standardized discriminant component, defined by a weighted sum of the original (standardized) parameters x_i (i=1,..,n) as shown in Equation 1.

$$z_j = \sum_{i=1}^{n} \alpha_{j,i} \cdot \frac{(x_i - \bar{x}_i)}{\sigma_i}\,; j = 1, \ldots, f;\, i = 1, \ldots, n \qquad [1]$$

By applying an iterative variant of Fisher's algorithm, the selection of a reduced subset of discriminant parameters can be optimized. In our study, the subjective preferences considered were grouped in two classes representing two evaluation levels. Five input parameters resulted significantly in the analyses performed and only the four most discriminant parameters were included as inputs in the subsequent fuzzy modeling process.

Interpretation of the Relationship Between Input Variables: Fuzzy Logic

Each fuzzy system links together several inputs to one output variable. The design of the fuzzy expert system consisted of the following steps: 1) fuzzification of model inputs and model output, 2) application of an inductive algorithm to identify the fuzzy qualitative model and interpret the user's preference, and 3) determination of a simplified rule-base enabling the model prediction.

Fuzzification

Input fuzzification converts the input and output variables into fuzzy variables, following the transformation defined by the membership functions. In this study, three fuzzy linguistic variables were considered to synthesize each numerical input value (**Figure 4a**). The term "low" has a membership function that is activated for values below the mean of each input variable, achieving full membership for values under the fifth percentile. The term "medium" has its maximum membership at the mean, with activation reaching until the percentile 5 and 95. Finally, the term "high," has a membership function that is activated for values above the mean, with full membership for values above the 95th percentile. Categorical inputs (when a user's opinion was regarded as an input variable) were directly fuzzified by introducing bell-shaped membership functions, because their wider spread rendered more consistent results (**Figure 4b**).

Output fuzzification was carried out in a similar way. Since the output response was entered by the subject in discrete levels (e.g., "satisfied" or "not satisfied"), bell-shaped membership functions were used. These were symmetrically distributed in the fuzzification window, as shown in **Figure 4c** for two output categories. Hence, the resulting fuzzification scheme was robust and problem-independent. It was not sensitive to outliers and could be computed in an automatic fashion.

Application of the Inductive Algorithm

The algorithm proposed and described in the Appendix is based on the concept of the adaptive fuzzy associative matrix introduced by Kosko (15). It works in

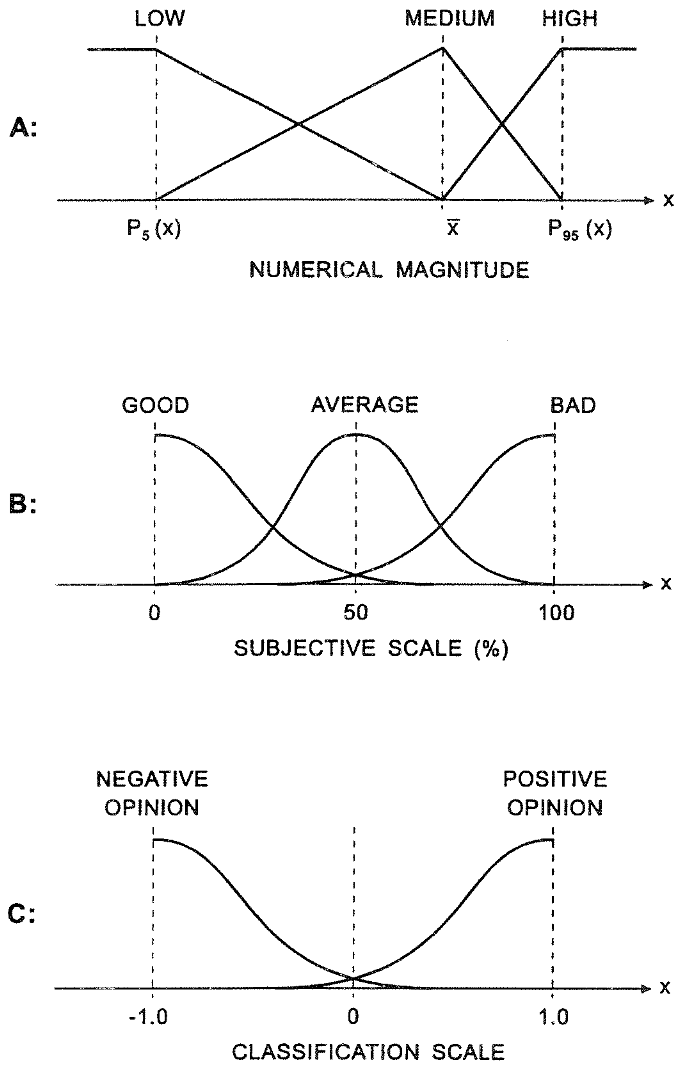

Figure 4.
(A) Input fuzzification of numerical parameters with triangular membership functions starting and ending at the mean value, the 5th, and the 95th percentile, respectively. (B) Input fuzzification of categorical variables by means of bell-shaped membership functions with symmetrical arrangement. (C) Output fuzzification of subjective dichotomous preferences by means of symmetrical bell-shaped functions.

an iterative manner by extracting functional rules from a data set provided during training. Here, this data set consisted of input parameters, which were measured from the wheelchair and output responses, that were asked of each user, so that a qualitative identification of the preferences was performed, as in a "black-box" approach.

At each iteration, a sort of "fuzzy contingency table" (the fuzzy associative matrix, FAM) was updated with new information linking the measured input vector to the desired output (see Appendix and Glossary). After the learning process, the resulting FAM represented the qualitative model of the system identified, with every rule characterized by a *recurrence* and a *reliability* score.

The recurrence is proportional to the relative frequency of use of the rule and reflects its importance within the training data set for explaining output responses. The more recurrent a rule, the more often this rule is required in order to account for the observations. The reliability indicates the certainty and correctness of a rule. The more reliable a rule, the less ambiguous and more accurate the description made by that rule. Both indices fall in the same real interval [0; 1], but are not necessarily bound together. For example, the rule "if the person is tall, the person is a man" might be an often recurring rule to explain observations of daily life, but its reliability is not high, because many women are also tall, thus constituting evidence against it.

The interpretation of this FAM was made by looking for clusters of similar behavior; that is, by trying to correlate a specific configuration of input fuzzy parameters with the same output response. These configurations thus represented an aggregation of individual fuzzy rules that could summarize the behavior of the system, and since the terminology employed for their formulation was easy for an expert to understand, their interpretation was also straightforward.

The time spent on the algorithm in the training process was relatively short: about 20 seconds for every user's preference analyzed using the MATLAB software package on a PC.

Rule-Base Determination and Model Validation

Fuzzy inference or prediction involves actualization of the fuzzy variables after fuzzification and application of the rule-base using the appropriate logical operators, resulting in different activation values for each rule. All fuzzy rules of the base are consulted and a combined averaged response is given (defuzzification), with each rule weighted by its reliability.

The number of rules in the rule-base was kept low and built with those fuzzy rules displaying maximum reliability and recurrence scores of the FAM.

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Table 2.
Significant aspects for the user's perception of global satisfaction.

Parameter	Acronym	# Positive Opinions	# Average Opinions	# Negative Opinions	SDC
Capability of adjusting the chair according to the demands of office job	ADJUST	45 (51%)	not asked	43 (49%)	0.57
Seating comfort	COMFORT	25 (29%)	54 (61%)	9 (10%)	0.46
Aesthetic outlook of wheelchair in user's opinion	LOOK	16 (18%)	50 (57%)	22 (25%)	0.37
Price-quality ratio in user's opinion	PRICEQUAL	10 (11%)	40 (46%)	38 (43%)	0.23
Wheelchair durability considering frequency of repairs	DURAB	29 (33%)	34 (39%)	25 (28%)	0.22

SDS = absolute value of the standardized discriminant coefficient.

Validation of the resulting fuzzy rule-base was carried out three times, each time preserving 25 percent of the samples for testing and employing the rest for fuzzy rule induction. Results were then averaged to assess the predictive behavior of the model.

RESULTS

First, the user's perception of global satisfaction with his/her chair was studied. Secondly, two functional aspects closely related to this perception were analyzed: adjustability and seating comfort. Other factors regarding mobility could have been investigated with the same methods, but were not within the scope of the field study (20).

Assessment of Global Satisfaction

The assessment of the users' global satisfaction was confined to two linguistic categories: totally satisfied or not satisfied. The answers "partially satisfied" and "not satisfied at all" were combined, with 36 percent of the users answering "satisfied" and 64 percent answering "not satisfied."

The input parameters selected and ordered in importance via FDA were all of subjective nature, preserving the original categorization in three linguistic terms, except for the adjustability, which was mentioned in only two options (see **Table 2**). The order of importance is as follows: adjustability for occupational purposes, seating comfort, aesthetic outlook, price/quality ratio, and durability. Only the first four were utilized in the fuzzy model. Fuzzification of these four input parameters was accomplished using bell-shaped membership functions, distributed as shown in **Figure 4b**. The outputs of the fuzzy model were:

- **Output 1 = User is satisfied.** The user expresses satisfaction with the wheelchair.
- **Output 2 = User is not satisfied.** The user is either partially or completely unsatisfied.

The algorithm for inductive fuzzy rule generation provided the FAM that is shown in **Figure 5.** Here, the theoretical five dimensions of the FAM were reduced to merely two by combining two input parameters in each axis and displaying the output of the expert system on the corresponding cell. The asterisks on both sides of the output represent the reliability score of the rule. The absence of an asterisk indicates that there is an insufficient reliability ($\alpha<0.2$); one asterisk = low reliability ($0.2\leq\alpha<0.5$); two asterisks = average reliability ($0.5\leq\alpha<0.8$); three asterisks = high reliability ($\alpha>0.8$). Finally, the shadowed cells designate those rules with the highest recurrence ($\rho>0.33\cdot\rho_{max}$).

By selecting the most recurring and reliable rules, three fuzzy statements, modeling the subject's perception of global satisfaction, were obtained (**Table 3.**) With these rules, an average of 81.0 percent of subject responses could be correctly predicted, with 4.5 percent indeterminations. An indeterminate response happened when none of the rules in the fuzzy rule-base got an activation superior to 0.01 per membership function. As a reference, FDA delivered a classification accuracy of 83.0 percent using the same qualitative input parameters codified in an ordinal scale.

Table 3.
Fuzzy rules expressing the user's perception of global satisfaction depending on the input preferences.

Rule	Recurrence ρ	Reliability α
IF ADJUST = GOOD & COMFORT = GOOD & LOOK = GOOD THEN OUTPUT 1 (user is satisfied)	0.28	0.98
IF ADJUST = BAD & COMFORT = NORMAL & LOOK ≠ NORMAL & PRICEQUAL = BAD THEN OUTPUT 2 (user is not satisfied)	0.20	1.0
IF ADJUST = BAD & COMFORT = BAD & LOOK = NORMAL & PRICEQUAL = BAD THEN OUTPUT 2 (user is not satisfied)	0.10	1.0

Table 4.
Wheelchair parts more frequently criticized for not being adjustable.

Parameter Assessed	Incidence
Back angle	48%
Seat width	42%
Seat height	41%
Armrest height	36%
Leg-to-seat surface angle	34%

Assessment on Functional Aspects

Adjustability of the wheelchair to the typical office tasks was the most discriminant factor associated with global user satisfaction. However, in the original questionnaire, this factor was assessed by asking the user to identify those parts that needed to be adjustable by responding in yes/no answers. Hence, the methods based on FDA and fuzzy logic were not applicable; instead a descriptive analysis with incidence rates is presented (**Table 4**). According to user opinion, the features that most urgently needed to be made adjustable were back angle, followed by seat width, seat height, and armrest height. With respect to the seat width, it should be noted that most wheelchairs are generally ordered to a specific size and are hardly ever adjustable.

Seating comfort was assessed for two subjective levels: the seat was either comfortable or it was not. Since a part of the interviewed users (27 out of 88) had removed their wheelchair armrests, only wheelchairs provided with armrests were considered for this analysis in order to avoid eventual bias. The categories "average comfort" and "uncomfortable" were combined, yielding a response of 17 (28 percent) comfortable and 44 (72 percent) average or not comfortable wheelchairs.

From all wheelchair dimensions shown in **Figure 2,** the five most significant input parameters selected via FDA are shown in **Table 5.** The first most discriminant variables, namely seat depth (SEATDPH), ground-to-back height (BACKHGT), ground-to-armrest height (ARMHGT), and seat angle (SEATANG), were used for fuzzy modeling. Fuzzification of these input parameters was accomplished using triangular membership functions, with corresponding maxima set at the 5th percentile (p-5), at the mean value, and at the 95th percentile (p-95). The possible outputs of the fuzzy model were:

- **Output 1 = Seat is comfortable.** The seat was perceived as being comfortable for office work.
- **Output 2 = Seat is not comfortable.** The seat was not perceived as being comfortable for office work.

The algorithm for inductive fuzzy rule generation provided the FAM that is shown in **Figure 6.** The display format is identical to **Figure 5.** By selecting the most recurrent and reliable rules, the fuzzy expressions of **Table 6** could be derived. With these rules, an average of 77.0 percent of subject responses could be correctly predicted, with no indeterminations. As a reference, FDA reached a score of 77.1 percent of correct hits using the five numerical input parameters listed in **Table 5.** Further functional aspects, such as aesthetic outlook and price-quality ratio, though actually relevant for the user's global satisfaction, were not analyzed in this context, because of the difficulty in correlating these aspects with objective dimensions of the wheelchair.

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Table 5.
Critical dimensions for seating comfort in office work.

Parameter	Acronym	Mean/cm	p-5/cm	p-95/cm	SDC
Seat depth (MW)	SEATDPH	42.2	38.0	46.0	0.59
Ground-to-back height (HW)	BACKHGT	84.6	78.1	89.8	0.45
Ground-to-armrest height (JW)	ARMHGT	70.2	64.2	72.0	0.38
Seat angle	SEATANG	4.8°	−1.4°	14.6°	0.36
Ground-to-footrest height (KW)	FOOTHGT	15.9	12.0	26.6	0.31

SDC = absolute value of the standardized discriminant coefficient.

Table 6.
Fuzzy rules explaining the seating comfort in dependence on the input dimensions.

Rule	Recurrence ρ	Reliability α
IF SEATDPH = HIGH & ARMHGT = HIGH & SEATANG = HIGH THEN OUTPUT 1 (seat is comfortable)	0.40	0.97
IF SEATDPH = HIGH & BACKHGT = MEDIUM & ARMHGT = HIGH & SEATANG = MEDIUM THEN OUTPUT 1 (seat is comfortable)	0.07	0.54
IF SEATDPH = LOW & BACKHGT = MEDIUM & ARMHGT = MEDIUM & SEATANG < HIGH THEN OUTPUT 2 (seat is not comfortable)	0.18	0.98
IF SEATDPH = MEDIUM & BACKHGT = HIGH & ARMHGT = MEDIUM & SEATANG = LOW THEN OUTPUT 2 (seat is not comfortable)	0.07	0.93
IF SEATDPH = MEDIUM & BACKHGT = MEDIUM & ARMHGT = HIGH & SEATANG = LOW THEN OUTPUT 2 (seat is not comfortable)	0.05	0.63

DISCUSSION

Design Criteria Based on User Preference

Although a great variability exists among individual physical factors and preferences, some general concepts are known to apply especially for active users (23,24). Interpretation of the user's preferences was made irrespective of the user's characteristics by regarding the recurrence and reliability score of each rule in the resulting FAM. The aim of the study was not to predict the user's behavior, but to validate the usefulness of the approach for the simplified interpretation of users' preferences. Additionally, some basic design recommendations for wheelchair manufacturers, though less relevant in this preliminary work, could be easily found and translated into common language.

In order to obtain significant results in classification and due to the limited sample size (88 interviewed subjects), users' responses were always divided in two categories (by eventually combining two response labels), thus improving the power of the experiment.

Global satisfaction was mainly affected by adjustability for occupational purposes, seating comfort, aesthetics, price-quality ratio, and durability. This presented a sound picture of the collective sampling: individuals working at offices with good functional capabilities, great concern for an ergonomic, attractive and well-adapted occupational environment, average income, and worries about frequent wheelchair repairs. It should be noted in this context that the ranking of preferences and evaluation of rules explaining a purely subjective opinion are not very common in previous

ASSESSMENT OF GLOBAL SATISFACTION				LOOK								
				GOOD			*NORMAL*			*BAD*		
				PRICEQUAL			PRICEQUAL			PRICEQUAL		
				GOOD	*NORMAL*	*BAD*	*GOOD*	*NORMAL*	*BAD*	*GOOD*	*NORMAL*	*BAD*
ADJUST	*GOOD*	COMFORT	*GOOD*	1 ***	1 ***	1 ***	1 ***	1	1 **	1 **	1 **	1 **
			NORMAL	1 ***	1	1 *	1	1 **	1	2 **	1	2
			BAD	1 ***	1 ***	1 *	1	1	2	2 ***	2 **	1 **
	BAD	COMFORT	*GOOD*	1 ***	1 ***	2	2	2 ***	2 ***	2 ***	1 ***	2 ***
			NORMAL	2 ***	2 ***	2 ***	2 ***	2 **	2 ***	2 ***	2 ***	2 ***
			BAD	2 ***	2 ***	2 ***	2 ***	2 ***	2 ***	2 ***	2 ***	2 ***

1: USER IS SATISFIED

2: USER IS NOT SATISFIED

Figure 5.
Fuzzy rule base for modeling the global satisfaction of the user. "1" denotes a positive opinion and "2" denotes a negative one. An increasing number of asterisks denotes a higher reliability.

literature, because of the difficult formal handling. FDA served to select and rank significant factors influencing the user's perception of global satisfaction and the inductive fuzzy algorithm allowed the interpretation of the internal dependencies between those factors.

The possibility of adjusting some parts of the chair—not frequently available in the sampled wheelchairs—resulted in the most important factor affecting the user's global satisfaction (SDC=0.57), followed by seating comfort (SDC=0.46), aesthetic outlook (SDC=0.37), price (SDC=0.23), and durability (SDC=0.22). Other relevant aspects of wheelchair design (e.g., ease of propulsion, maneuverability, safety) did not appear in the study as key factors, because the questionnaire was focused on working conditions and, on the other hand, standard wheelchairs were actually less prepared for their adaptation to the workplace than to the restoration of mobility.

The main conclusions that could be drawn from the FAM in **Figure 5** are:

1. Poor adjustability of the wheelchair was the most common reason to dislike the chair. If this negative aspect appeared together with average or poor seating comfort, the chair was usually rejected. Wheelchairs that were not adjustable, should at least be inexpensive and attractive in order to be acceptable to the user.
2. Easily adjustable chairs, with a seat that was also comfortable, were generally well-considered by the user. However, the esthetic outlook played an important role in this group. Inexpensive chairs with an unpleasant appearance and average or poor seating comfort tended to be evaluated poorly by the user. Expensive chairs usually led to a less reliable user response.

A reduced set of fuzzy rules rendered more than 80 percent of correct predictions, which confirms the graphical impression of the FAM showing two general patterns of behavior: a group of satisfied users with easily adjustable, comfortable, and attractive wheelchairs, and another category of users with hardly adjustable and expensive wheelchairs.

With regard to adjustability, it was remarkable that the parts of the wheelchair that were strongly demanded to be adjustable were precisely those parts that actually should be adjustable in a well-designed conventional

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ASSESSMENT OF SEATING COMFORT				ARMHGT								
				LOW			*MEDIUM*			*HIGH*		
				SEATANG			SEATANG			SEATANG		
				LOW	*MEDIUM*	*HIGH*	*LOW*	*MEDIUM*	*HIGH*	*LOW*	*MEDIUM*	*HIGH*
SEATDPH	*LOW*	BACKHGT	*LOW*	2 *	2 **	1	2 **	2 ***	2 ***	1	1 **	1 ***
			MEDIUM	2 **	2 ***	2 ***	2 ***	2 ***	2 ***	1 *	1 *	1 **
			HIGH	2 ***	2 ***	2	2 ***	2 ***	2 ***	2 ***	2 **	2
	MEDIUM	BACKHGT	*LOW*	1	1	1 *	1 *	2	2 **	2 **	2	1 *
			MEDIUM	2 *	2	1 *	2 *	2	2 *	2 **	1	2 *
			HIGH	2 ***	2 ***	1 **	2 ***	2 *	2 *	2 ***	2	2 *
	HIGH	BACKHGT	*LOW*	1 ***	1 ***	2	1 ***	1 ***	2 ***	1 ***	1 ***	1 ***
			MEDIUM	1	1 **	1 **	2	1	2	1	1 **	1 ***
			HIGH	2 ***	2 *	1 ***	2 **	2	1	2 ***	2 **	1 ***

1: SEAT IS COMFORTABLE

2: SEAT IS NOT COMFORTABLE

Figure 6.
Fuzzy rule base for modeling user perception of seating comfort. "1" denotes a positive opinion and "2" denotes a negative one. An increasing number of asterisks denotes a higher reliability.

office chair (e.g., back angle, seat height, backrest position, and armrest height). This would confirm that ergonomic aspects related to the wheelchair-office configuration are not yet adequately solved in many situations. Since these were the problems that active wheelchair users who worked in offices judged even more relevant than other aspects related to mobility, further investigation on these topics seems advisable.

With reference to seating comfort, 72 percent of the subjects using wheelchairs with armrests at the workplace considered the chair uncomfortable. The parameters that played a key role were the seat depth, with special significance (SDC=0.59); the ground-to-back height (SDC=0.45); the ground-to-armrest height (SDC=0.38); the seat angle (SDC=0.36); and the ground-to-footrest height (SDC=0.31). From these results, an appropriate match between seat depth, back height, and seat angle seems essential in order to maintain a proper posture while working. In addition, adjustment of the armrest and footrest height appears to be critical for preserving a sufficient accessibility to the office furniture.

A close look at the FAM in **Figure 6** revealed that seating discomfort while working was typically caused by the following features of the wheelchair:

- A short seat with armrests that are not high enough; probably due to a deficient pressure distribution
- A high backrest with a seat angle that is not steep enough.

In analogy, a perception of seating comfort while working was the result of a wheelchair with the following configuration:

- A deep seat and a low backrest, which would probably lead to a better load distribution and easier upper limb mobility

- A deep seat and a steep seat angle, provided that armrest height was properly set.

If we consider armrest height and seat angle as being adjustable, some recommendations could be suggested from the FAM:

- For shorter seats and lower backrests, a sufficient armrest height should be selected
- For an average seat depth, a high seat angle and a lower armrest height are better perceived by the user
- For deep seats and higher backrests, seat angle should be set at a steep position and armrest height should be adequately chosen for that angle.

Two clusters of reliable and frequent rules could be distinguished: 1) a group of comfortable wheelchairs with deep seats, lower backrests, high armrests, and steep seat angles; and 2) a class of uncomfortable chairs with shorter seats, average armrest height, and lower seat angles. The few fuzzy rules extracted could correctly model the subject's opinion in 77 percent of the cases.

Translation of the linguistic terms "low," "medium," and "high" into useful design criteria can be easily accomplished by considering the fuzzification scheme of the input dimensions and knowing the definition points of each fuzzy set: 5th percentile, mean value, and 95th percentile. For example, a typical high seat angle for the sampled population would be any inclination above 9.7° (the intersection of fuzzy sets medium and high for this variable).

Finally, it should be noted that further design criteria could have been derived from the FAMs by looking at specific user-wheelchair configurations; that is, by studying fuzzy rules in appropriate cell positions. Extra dimensional parameters could have been added for modeling each subjective assessment as well. This might have led to new insights about optimal design. However, our intention was to keep complexity in limits, showing a preliminary application of fuzzy techniques. Furthermore, it should be also stated that generalization of these results is only possible with great caution, because the collective analyzed is hardly representative of the global population of wheelchair users.

Proposed Methodology

The results presented show that introduction of fuzzy logic in a field where subjective opinions play a primary role constitutes an interesting step for adapting mathematical tools to the human way of reasoning; thus providing an efficient way to cope with imperfect information (25).

The inductive learning algorithm proposed was developed starting from several ideas published in the fuzzy literature (14,15,26). Our major contribution was the introduction of two essential concepts: recurrence and reliability of the fuzzy rules. The algorithm is universal and can be employed for modeling any system in a qualitative fashion in numerous domains of application, either of subjective feelings or of objective measurements.

The accuracy of the prediction of the derived rule-bases was similar or even better than the classification accuracy of FDA. Variability of the dimensions and functional capabilities of the subject could explain why some preferences were misclassified. If we take into account that fuzzification always implies a certain loss of information (quantitative are transformed into qualitative variables), our results demonstrate not only the interpretability but also the discrimination potential of the introduced methods.

The process for obtaining a fuzzy rule-base describing user preference is completely automatic and problem-independent. No arbitrariness is introduced during this process, since membership functions are defined in a standard way from statistical landmark values (percentiles and means), input parameters are selected from among a multiple set of measured magnitudes by means of the linear FDA, and the FAM is computed automatically with the help of the algorithm. Moreover, working with the techniques proposed is simple even for people not familiar with computers and statistics, since every design stage can be efficiently automated. The interpretation of the resulting FAM is straightforward as well, because the concepts involved (linguistic terms, fuzzy rules, recurrence, reliability) are comprehensible and not difficult to translate into practical design criteria.

In this sense, it is important to remark that, although previous knowledge of the concepts involved is necessary in order to sort out the significant input parameters with FDA, the selection of fuzzy rules and the subsequent interpretation of the users' preferences can be performed without resort to the *a priori* knowledge of the human expert. Obviously, this expertise is helpful in speeding up the interpretation and making it more profitable.

Finally, the mapping of linguistic concepts into membership functions could be optimized by means of strategies based on genetic algorithms or neural networks. This sort of technique could better match the qualitative model to the observations, thus reducing the estimation error.

CONCLUSIONS

The introduced methods consisted of the fuzzification of the parameters featuring the user-wheelchair interface in order to discover the rules that modeled preferences of a collective of users interviewed via a questionnaire aimed at studying the adaptation of the wheelchair to the office workplace. Fuzzy rules were automatically determined by using a self-developed inductive algorithm that rendered an associative matrix with every rule characterized by its recurrence and reliability score.

The aim of the derived rule-bases was not to predict the behavior of the user in new settings (other subjects and other wheelchairs), but to extract the implicit subjective criteria within the collected data that played a role in the user's perception of global satisfaction and seating comfort. For this, previous expertise was helpful for selecting potentially significant input dimensions and for interpreting the resulting rule-base.

The results reveal a mismatch between actual performance of standard wheelchairs and the requirements of work in an office in the sampled collective. Functional aspects criticized by the users agree with essential needs also detected in office workers with no disability.

The proposed methods provide a flexible tool for wheelchair design based on questionnaires that are user-friendly. The concept could be also extended to the area of prescription by considering user characteristics in the fuzzy model.

APPENDIX

Algorithm for Inductive Fuzzy Rule Generation

The problem of modeling an unknown system by means of fuzzy rules (in our case, the opinion of a person using a wheelchair) can be considered as a problem of system identification, with several measured input variables $x_i(i=1,..n)$ feeding the system (black box) and with a unique response y representing the response of the system. The fuzzy rule-base modeling this system sets up a qualitative model, because no numerical differential equations are used; instead rules employing linguistic terms are used. Input and output variables may be of an arbitrary nature (quantitative, qualitative, physical, or psychological).

When all relevant input variables have been determined (for instance, by means of FDA), input fuzzification must be specified and the rule-base generated. The first task must be solved by the designer using his/her *a priori* knowledge for defining appropriate membership functions; in this article, one method is proposed, based on the mean value, and the 5th and 95th percentile.

The second task is much more complex and, in many practical cases, is carried out by asking a human expert to formulate his/her knowledge in the form of heuristic rules (using his/her habitual language) and subsequently converting those rules into a fuzzy rule-base. However, this procedure of knowledge acquisition is prone to the subjectivity of the domain expert and is sometimes not affordable. We propose an alternative method that stems from the original formulation of the adaptive FAM by Kosko (15).

Given a fuzzy system with n quantitative or qualitative input variables x_i $(i=1,..,n)$, each of them distributed into f_i fuzzy sets (linguistic descriptives), and one quantitative or qualitative output variable y again divided into f_{n+1} fuzzy sets, the total number of dimensions in the FAM is $n+1$, being C_0 the global set of possible cells of that matrix. The total number R_0 of possible fuzzy rules is equal to the number of cells in the FAM.

$$R_0 = f_1 \cdot f_2 \cdots f_n \cdot f_{n+1} \qquad \text{[A-1]}$$

As an illustration, we could think about an expert system for evaluating the degree of comfort of wheelchair seats given its depth and inclination. Imagine that we had these two inputs (n=2), each one partitioned into three fuzzy sets (f_1=f_2=3), for example, low, medium, and high, and the output again into two fuzzy sets (f_3=2), and "seat uncomfortable" and "seat comfortable." The total number of FAM dimensions would be three and the amount of possible fuzzy rules R_0=18. The first rule in the upper left corner of the FAM would correspond to the conjunction of the first fuzzy set per variable, reading: "When the seat's depth is *low* and the seat's inclination is *low* the seat is *uncomfortable*."

In order to automatically obtain the main rules governing the system, it is necessary to monitor the recurrence ρ_i of each fuzzy rule (i=1,..,R_0), as the system intends to reproduce known output responses from given input values. For each real observation (x_k; y_k), a fuzzy rule subset $C_R \subset C_0$ becomes active in order to explain the output. The activation δ_i of each rule is the result of the logical AND of the antecedent's μ(ANT) and the consequent's activation μ(CONS). No such activation is computed when the fuzzy implication is not fulfilled, that is, when the membership function value of the rule's consequent is lower than the rule's antecedent value. Computing the AND operator with the product, the rule's activation is defined as follows:

$$\delta_{i,k} = \begin{cases} \mu(ANT_{i,k}) \cdot \mu(CONS_{i,k}) \; ; \; \mu(ANT_{i,k}) \leq \mu(CONS_{i,k}) \\ 0 \qquad\qquad\qquad\qquad\quad ; \; \mu(ANT_{i,k}) > \mu(CONS_{i,k}) \end{cases} \qquad \text{[A-2]}$$

The activation counter ρ_i is incremented at each step k (new observation) by the amount of activation of the rule δ_i. Hence, after each iteration the active rules $r \subset C_R$ are updated according to the following law:

$$\Delta\rho_{i,k} = \begin{cases} \delta_{i,k} \; ; \; \forall\, r_i \in C_R \\ 0 \; ; \; \forall\, r_i \notin C_R \end{cases} \qquad \rho_{i,k} = \rho_{i,k} + \Delta\rho_{i,k} \qquad \text{[A-3]}$$

Once all observations have been presented (x_k; y_k), k=1,..,N, an activation pattern is obtained for every cell (rule) in the FAM. We then normalize each activation by the total sum of activations D, resulting in a relative rule activation or **recurrence** ρi with values in the interval [0; 1]:

$$D = \sum_i \rho_i \quad ; \bar{\rho}_i = \frac{\rho_i}{D} \qquad \text{[A-4]}$$

Obviously not all rules with the same antecedent can be valid at the same time, because this would imply a logical contradiction. From the f_{n+1} rules responding to the same stimulus x_k, merely one is valid. Therefore, only that rule among the rule set with the same antecedents whose relative activation (recurrence) is maximal is taken into consideration. In any case, the significance of that rule clearly depends on the dispersion of activity among the f_{n+1} rules responding to the same antecedent. The more disperse, the less reliable the selected rule will be.

Reliability α_i of a rule is expressed by the difference between the highest recurrence and the second highest recurrence, normalized by the sum of recurrences for the same antecedent. The reliability has values in the interval [0; 1] shown in formula A-5.

$$\alpha_i = \frac{\bar{\rho}_m - \bar{\rho}_s}{\sum_{j=1}^{f_{n+1}} \rho_j} \; ; \; \rho_m \geq \rho_s \geq \rho_j \; ; \; i = 1, \ldots, R_0 \qquad \text{[A-5]}$$

Therefore, each rule is characterized by two features: its recurrence $\bar{\rho}_i$ and its reliability α_i in the training data set. Once the FAM matrix has been calculated, it is straightforward to read those rules that better synthesize the phenomenon under study. As a general recommendation, we select those rules that exhibit the greatest recurrence together with the maximum reliability, avoiding an excessive number in order to facilitate the subsequent interpretation. After selecting those rules, a rule compression by means of fuzzy logical laws can be carried out (association, distribution, and so forth).

To conclude, a final sentence should be devoted to classification problems such as the one in this paper. Here the output does not need to be defuzzified into a continuous value; instead, group membership probabilities are desired. After defining each class as a fuzzy set, we can consider the membership to these sets proportional to the probability of pertinence to that group computed by the expert system. In this case, normalization of the activities (recurrence) of the rule will be based on the *a priori* probabilities defined for each group. Dividing each activation counter by the total activation sum D, implies the sampling of the population with its actual group proportions. In laboratory experiments, however, the number of samples per category often is not equivalent to the expected probabilities. If we want to assume equal *a priori* probabilities for every class, we should then normalize each rule affecting one output fuzzy set (a classification class) by the total activation sum D_k of this output fuzzy set (k=1,..,K), with K being the total number of possible classes.

GLOSSARY

Qualitative model. A simplified description of a complex reality that uses qualitative knowledge (knowledge that cannot be expressed by means of numerical terms) in order to predict a physical system's behavior.

Fuzzy associative matrix (FAM). A representation of a rule-base in conjunctive form; fuzzy rules combine input variables by means of logical ANDs. The FAM forms a hypercube with as many dimensions as input variables, together with one output dimension. Each cell stands for a hypothetical rule. The number of cells in each axis depends on the number of fuzzy partitions (linguistic variables) defined for that input.

Fuzzy rule. An IF-THEN rule that explains observations or predicts a behavior in a qualitative way, built up with fuzzy linguistic variables connected by logical operators (AND, OR, NOT, etc.). A rule consists of an antecedent expression and a consequent expression. The latter can only be true if the antecedent is also true.

Fuzzy rule-base. A set of fuzzy rules constituting a qualitative model of the behavior of a physical system.

Fuzzy rule activation. The activation of a rule within a rule-base depends on its explanatory or predictive power. It is proportional to the degree of truth of the antecedent when the rule is used for prediction, and is proportional to the degree of truth both of antecedent and of consequent when the rule is used to explain an input/output observation. Faced with an specific input, only a subset of the rules forming the base is activated and just this subset accounts for the given response.

Fuzzy implication. Analogous to Boolean implication, saying that the degree of truth of a rule's consequent can never be less than the degree of truth of the corresponding antecedent. In fact, this is the definition of a rule.

Input parameter vector. The vector formed by all significant parameters featuring a wheelchair-user configuration. The parameters can be measured and the whole set of measured parameter vectors builds up the observation matrix.

Output response. A subjective judgment, expressed in categorical linguistic terms, about a specific feature of the wheelchair-user configuration.

Training data set. A set of parameter vectors labeled according to their corresponding output response and used to train the fuzzy expert system.

Validation data set. A set of parameter vectors that is independently and randomly chosen from the whole observation matrix and used to test the performance of the fuzzy expert system.

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