Journal of

Rehabilitation Research and Development

VOL. 35 NO. 2 1998

A Peer-Reviewed Journal

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Library of Congress Catalog Card No. 6.60273
ISSN 0007-506X

The *Journal of Rehabilitation Research and Development* is a quarterly publication of the Rehabilitation Research and Development Service, Department of Veterans Affairs.

The opinions of contributors are their own and are not necessarily those of the Department of Veterans Affairs.

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The *Journal of Rehabilitation Research and Development* and its supplements are printed on acid-free paper, as of Vol. 25 No. 2 (Spring 1988).

Department of Veterans Affairs

Journal of Rehabilitation Research and Development
June 1998, Vol. 35, No. 2

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LETTERS TO THE EDITOR

To the Editor:

Finite Element Modelling of Bulk Muscular Tissue

W.M.Vannah and D.S.Childress (*Journal of Rehabilitation Research and Development,* 1996;33;239–52) report a finite element study of bulk muscular tissue *in vivo.*

The non-linear load/extension (or stress/strain) behaviour of the tissue is modelled as a hyperelastic material, which has a Strain Energy Function (SEF), which is a function of the first and second extension ratio invariants (I_1; I_2) and is given by:

$$W=c_{10}\,(I_1-3) + c_{01}\,(I_2-3) + c_{11}\,(I_1-3)\,(I_2-3) \qquad [1]$$

where c_{ij} are constants.

The soft body tissues are known to be nonlinear, anisotropic, and time dependent. The elastic SEF in Equation 1 is isotropic because it depends only on the extension ratio invariants and incompressible because it is not a function of I_3. These limitations are questionable although, given the complexity that arises if they are violated, it would appear justifiable to use them as a first step in modelling the mechanical nonlinearity of the tissues.

The soft tissues can undergo large deformations when tested in simple tension or compression and this produces large changes in the cross-sectional area of the test specimen. Under these circumstances, the usual engineering stress (s), which is referred to the original cross-sectional area of the test specimen, may be very different from the Cauchy stress, which is referred to the cross-sectional area of the deformed specimen. It is difficult to measure the changing cross-sectional area experimentally and results of simple tension and compression tests carried out on tissues are generally reported in the literature using either the engineering stress or the load (which is proportional to the engineering stress). Similar problems arise in defining an appropriate measure of strain. Reports in the literature usually used either engineering strain (e), or extension ratio (λ = 1 + e) if the results are reported within an SEF context.

It is difficult to compare published results with Figure 10 of Vannah and Childress because the latter use Cauchy stress and logarithmatic strain.

The engineering stress/extension ratio relationship for a material with an SEF which is a function of I_1 and I_2 is given by (1,2):

$$s=2\left(\lambda-\frac{1}{\lambda^2}\right)\left[\frac{\vartheta\varphi}{\vartheta I_1}+\frac{1}{\lambda}\frac{\vartheta\varphi}{\vartheta I_2}\right] \qquad [2]$$

For the SEF defined in Equation 1, this becomes:

$$s=2\left(\lambda-\frac{1}{\lambda^2}\right)\left\{C_{10}+C_{11}(I_1-3)+\frac{1}{\lambda}[C_{01}+C_{11}(I_2-3)]\right\} \quad [3]$$

Figure 1 shows the stress-extension ratio behaviour predicted by Equation 3 utilising the parameters given by Vannah and Childress of c_{10} = 2.6 kPa, c_{11} = 5.7 kPa and $c_{01} = c_{10}/4$.

The curve shown in **Figure 1** is qualitatively plausible as a description of the compressive and tensile behaviour of muscle but there are both qualitative and quantitative limitations.

The transition from the initial highly compliant phase of behaviour to the intermediate stiffer region is generally very much more abrupt than that shown in **Figure 1** and occurs at much lower extension ratios. To highlight this difference, **Figure 1** includes experimental results we have obtained from compression tests of excised muscle. In addition, the tension curve shown in **Figure 1** has a change of modulus between low and high extensions, which is considerably less than that found experimentally. These discrepancies may have arisen either because the general form of the SEF in Equation 1 is inappropriate or the parameter values are poorly chosen.

Vannah and Childress use the relationship $c_{01} = c_{10}/4$, saying that it was a "customary assumption" and quote Oden (3) and Ogden (4) in support. However, neither reference discusses materials with the stored energy function given by Equation 1. Oden gives stress analysis examples based on Mooney materials for which W depends only on c_{10} and c_{01}. In some examples, they use $c_{01} = c_{10}/4$, although others use different ratios.

We believe that Vannah and Childress have been excessively restrictive in the assumption they make about the ratio of the two leading terms in Equation 1 and that this restriction is unjustified because the examples for Mooney materials are inconsistent and cannot be extrapolated to Equation 1.

We have used the SEF given in Equation 1 to model the compressive properties of muscle displayed in **Figure 1,** using a least squares method. The results in **Figure 2** identify the best-fit values of the three constants. The agreement between predicted and experimental results in compression is impressive although the predicted tensile behaviour has the problems outlined about in relation to **Figure 1**.

The value of the constants produced by the least squares procedure and used in **Figure 2** below are c_{01}=0.0863 kPa; c_{10}= −0.0896 kPa; c_{11}= −0.158 kPa.

The appearance of negative constants is somewhat surprising and, apparently, unprecedented. Nevertheless, the SEF containing these negative constants seems to be physically acceptable.

We conclude that the three-term SEF proposed by Vannah and Childress provides a good description of the mechanical properties of muscle tested in simple compression. The tensile behaviour, although not ideal, is also acceptable. More work needs to be carried out,

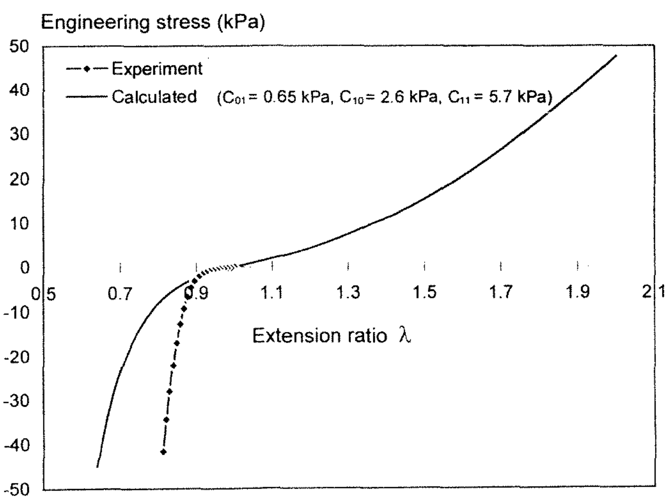

Figure 1.
Extension ratio-engineering stress relationships proposed by Vannah and Childress ("calculated") and experimental results obtained by compression of excised muscle.

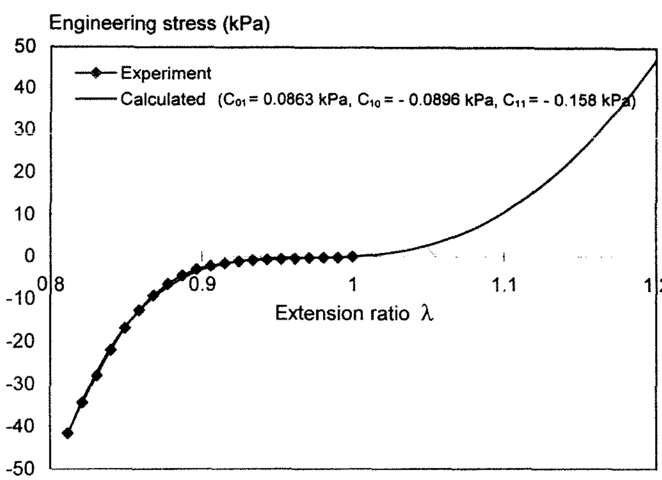

Figure 2.
In uniaxial tension and compression of a tissue with strain energy constants c_{01}=0.0863 kPa; c_{10}= −0.0896 kPa; c_{11}= −0.158 kPa "calculated" – (continuous line) and experimental results obtained from compression tests on excised muscle.

however, to assess the usefulness of the SEF in other modes of deformation, particularly in multiaxial states of stress or strain.

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