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Wheelchair caster shimmy II: Damping

James J. Kauzlarich, PhD; Theodore E. Bruning III; John G. Thacker, PhD
University of Virginia, Mechanical Engineering Department, Charlottesville, VA; Compaq Computer Corporation, South Colorado Springs, CO

Abstract—The theory of shimmy damping is investigated including tire friction, spindle bearing friction, and hydraulic damping. A new theoretical improvement in hydraulic damping is presented. Experimental results are presented along with a discussion concerning the limitations due to the approximations used in the theory. The basic theory of wheelchair caster shimmy was published by the authors in 1984, and an examination of the sources of shimmy damping is corrected and updated in this paper.

Key words: *caster design, casters, damping, shimmy, wheelchair casters.*

INTRODUCTION

Self-excited vibration is one of the most interesting topics in the field of vibrations and is the science governing caster wheel shimmy. Caster wheel shimmy can be experienced in everyday equipment, such as wheelchairs, grocery carts, gurneys, teacarts, and the like, and is universally recognized. Self-excited vibration is characterized by vibration that is produced by the motion of the system (e.g., wheelchair speed) itself. The flutter due to the motion of turbine blades or aircraft wings is a good example of this instability. In addition, machine tool chatter, internal flow-induced vibration of piping, and cross flow-induced vibration of wires and structures are treated under this topic in modern vibration texts (1–3).

In 1984, the authors presented a paper on the subject of wheelchair caster shimmy and turning resistance (4), with both theory and experimental data. Since that time, manufacturers have considered several methods for shimmy prevention, and the problem of shimmy prevention is well in hand. However, the theory behind shimmy prevention is not well known. This paper presents applications of the basic theory to show how shimmy prevention works in ultra-light and powered wheelchairs. In addition, the theory leads one to new designs. The theory explains why a trial and error design may or may not work, and suggests possible new solutions.

Theory Revisited

A simplified model of the caster wheel is shown in **Figure 1**. Here, the free body diagram of forces and moments can be solved to determine the bearing loads as a function of the applied load on the lower bearing, the trail of the wheel, and the distance between the bearings. The center of gravity of the caster wheel system is shown on the fork.

Address all correspondence and requests for reprints to: James J. Kauzlarich, PhD, University of Virginia, Mechanical Engineering Department, Charlottesville, VA 22903-2442; email: jjk@virginia.edu.

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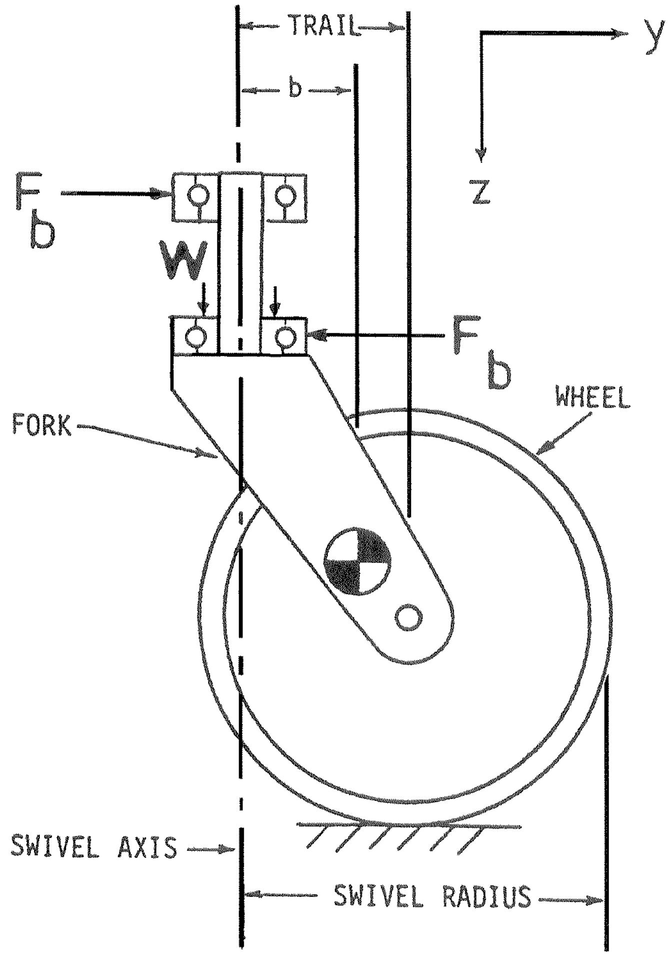

Figure 1.
Caster-wheel schematic. F_b, force on bearing; W, load; b, dimensions.

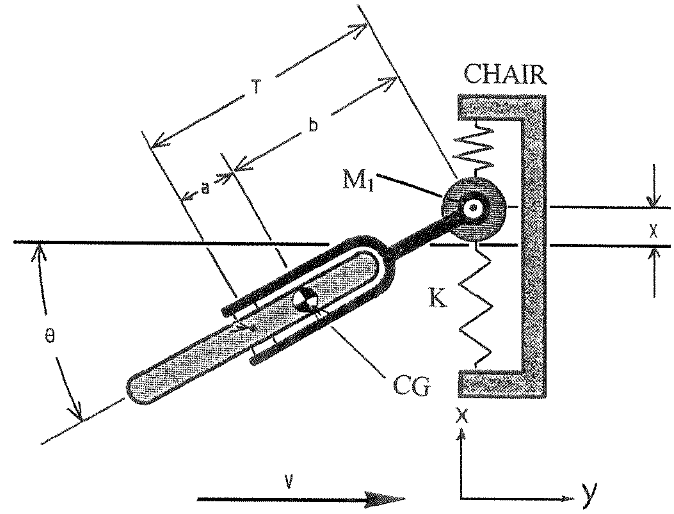

Figure 2.
Wheelchair caster model. M_1, mass of spindle bearing house; K, spring constant; T, caster trail; a,b, dimensions; V, wheelchair velocity; θ, angular displacement; CG, center of gravity.

The dynamic model assumed for caster shimmy is given in **Figure 2**. This two-degree-of-freedom model is minimally sufficient, but not complete, for wheelchair caster shimmy analysis. For example, treating the frame as a rigid member is not satisfactory but simplifies the problem considerably. Grooved tires that were treated in the 1984 paper will not be considered in this paper. For more information on this topic, see the Kauzlarich and McLaurin patent (5).

The majority of caster wheels use a vertical spindle. Mochel (6) investigated the effect of tilting the spindle axis, and concluded that camber and forward tilting were undesirable, and backward tilting only increased the effective trail.

The results of the previously published theory analysis by Kauzlarich, et al. (4), are summarized by the following equations. The viscous damping coefficient (C_D) is related to the trail (T), critical shimmy velocity of the wheelchair (V_c), and wheel mass moment of inertia (I_w) about a wheel diameter, as derived in (4, Equation 20B), and is reproduced in Equation 1, below.

$$\sum C_D = \frac{V_c}{T} I_w \qquad [1]$$

Equation 1 is a simplified version of a full analysis, but it was shown, by Morland, to be applicable to aircraft caster shimmy problems when the torsional spring rate of the damper linkage is infinity (7, Equation 6). In the design of wheelchair casters, the effective damper linkage is the steel fork, which is torsionally very stiff (exceptions occur for 'frog legs' spring forks), and Morland's result is identical with Equation 1 above.

The steady state critical angular frequency at onset of shimmy (ω_c), from (4, Equation 15B), is found to be a function of trail (T), wheel mass moment of inertia (I_w) about a wheel diameter, spring rate from spindle to chair (K), and the mass of the fork and spindle bearing housing (N); see Equation 2 below.

$$\omega_c^2 = \frac{KT^2}{I_w + NT^2} \qquad [2]$$

The steady state critical angular amplitude of shimmy (θ_a) from (4, Equation 6C) is shown to be a function of the initial deflection of the caster wheel (θ_0), the critical velocity of the wheelchair (V_c), the critical shimmy frequency (ω_c), and the trail (T), as shown in Equation 3,

$$\frac{\theta_a}{\theta_0}=\frac{V_c}{\omega_c T} \qquad [3]$$

where the initial condition of angular velocity in (4, Equation 6C) is set as $\dot{\theta}=0$, as this was found experimentally to be a realistic model for typical wheelchair operation.

There were a number of assumptions applied to the shimmy model used by Kauzlarich, et al. (4) in the original derivation of Equations 1–3. A better understanding of the limitations of this theory is the objective of the next section.

RESULTS

The test results were obtained with a specially built wheelchair tethered on a treadmill designed for these tests. The caster fork for the tests was constructed to allow easy changing of the trail. The fork spindle was supported by two standard, sealed ball bearings (bearing pitch diameter, d_m=19.5 mm) using low viscosity oil, and with no other damping mechanism for the original tests. Test results for an added frictional damper and an added hydraulic damper will be discussed later. Using the test fork or commercial rigid forks gave essentially the same results (8); spring forks were not tested.

Figure 3 is a typical plot of test results for a particular tire, caster load, and trail. An important theoretical observation (Equation 3) is that the critical (onset) velocity of shimmy is a function of the initial angular deflection of the caster wheel (θ_0). From **Figure 3a,** the initial angular deflection was 30º and V_c=1.20 m/s, but from **Figure 3b** the initial angular deflection was 25º and V_c=1.33 m/s. In order to establish the functionality of the original deflection on critical shimmy velocity, it is necessary to consider the damping associated with the caster wheel of these tests. In the original paper, it was assumed that the only frictional damping was associated with the spindle bearings. However, these bearings have a very low torsional friction mainly due to the rubbing of the seals (measurements gave a Coulumb friction torque, or frictional dampening moment, M_f, equal to 0.005 N-m per bearing), and are essentially independent of the load on the bearing.

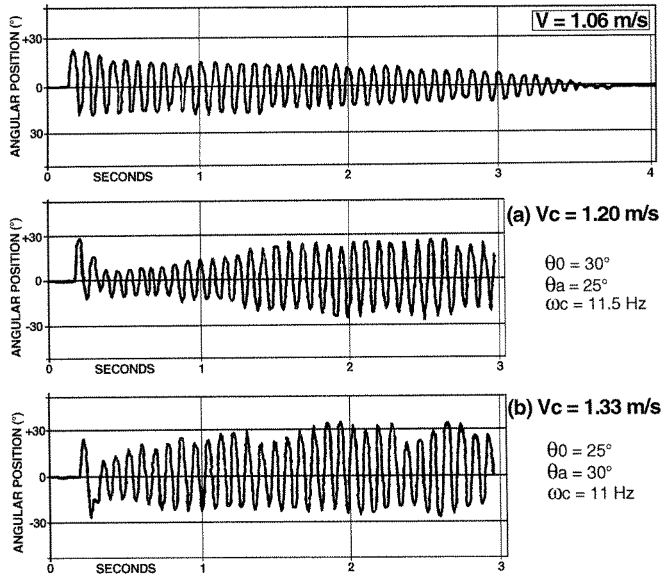

Figure 3.
Shimmy trace (E&J Mod I 8 in PU tire; I_w = 0.00197 kg-m^2; Trail = 0.0381 m (1.5 in); Load = 175 N (35 lbs). I_w, mass moment of inertia of wheel about a wheel diameter; V, wheelchair velocity; V_c, shimmy critical velocity; θ_0, initial angular displacement; θ_a, angular amplitude; ω_c, shimmy critical frequency.

The only other source of frictional damping is at the contact of the tire with the floor. As the caster begins to shimmy, the torsional friction of the tire with the floor as the tire oscillates is the main source of frictional damping in the tests, and is load-dependent. Assuming only tire/floor frictional damping, the critical shimmy equation derived originally (4, Equation 6) results in the following governing equation for the onset of shimmy, where it is assumed that the tire is not grooved, i.e., (4, Equation 7), and is reproduced in Equation 4 below.

$$\frac{V_c}{T}=\frac{\sqrt{4M_f}}{\pi\theta_0 I_w} \qquad [4]$$

A plot of critical shimmy speed versus trail for a test caster wheel under a fixed load is given in **Figure 4**. Here it appears that Equation 4 is a good model for shimmy. When the initial torsional deflection was 30º as shown by open squares, the plot is a straight line in agreement with Equation 4, and as the initial torsional deflection is reduced to 23º the critical shimmy velocity increases as the square root of $1/\theta_0$. Using Equation 4, it is possible to estimate the effect of varying the initial torsional deflection of the caster wheel on shimmy speed, and this is

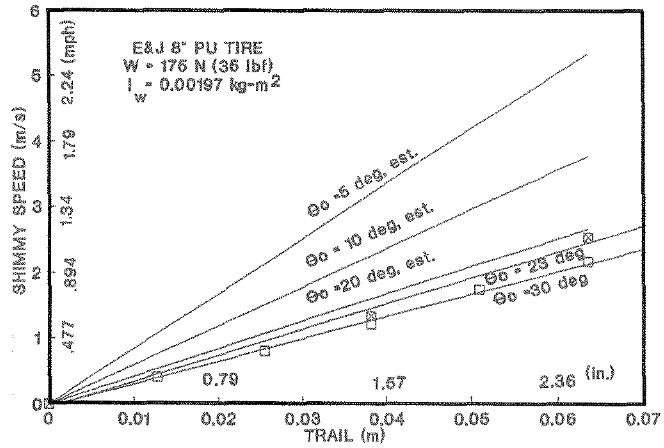

Figure 4.
Shimmy speed versus caster trail. W, load; I_w, mass moment of inertia of wheel about a wheel diameter; θ0, initial angular displacement.

shown in **Figure 4**. During testing, the initial deflection of the caster was controlled by the operator. Measurements of the random deflections of the caster wheel on the test treadmill were found to be approximately 5–10º (6).

A further comparison of theory with experiment is shown in **Figure 5**. The results show that the theory predicts the critical angular frequency correctly over the entire range of caster trail (T) tested. However, the theory does not predict the critical amplitude of shimmy properly below a trail of about 0.04 m (1.57 in). One of the assumptions in the derivation of the theory is the "small angle assumption" where tan θ=sine θ=θ. One can easily show that this assumption is significantly in error above θ=30º. Most wheelchair casters (except sports chairs) have a trail about 0.04 m (1.57 in) where $\theta_a \leq 30º$, and where the small angle assumption is applicable. Using the test data, and Equation 2, it was determined that the spring rate between caster and chair was $K=1.5\times10^4$ N/m for the test wheelchair system.

Caster wheel loading varies from 30–40 percent of laden weight normally, with very active users going as low as 5 percent of laden weight. The tests showed an interesting low load stability not predicted by the simple theory. When the load on a caster was reduced to about 100 N, it was found that the critical shimmy speed jumped up, as shown in **Figure 6**. At 40 percent of laden weight on the casters, an empty wheelchair of 175 N (39 lbs) would experience a load of 35 N per caster, and would appear to be shimmy stable (see **Figure 6**)—giving a false indication of shimmy characteristics under loaded wheelchair conditions. **Figure 6** also shows that the shimmy speed increases linearly with load above a load of 100 N, indicating that the frictional damping moment (M_f) is changing with load in Equation 4. This effect is discussed in the next section.

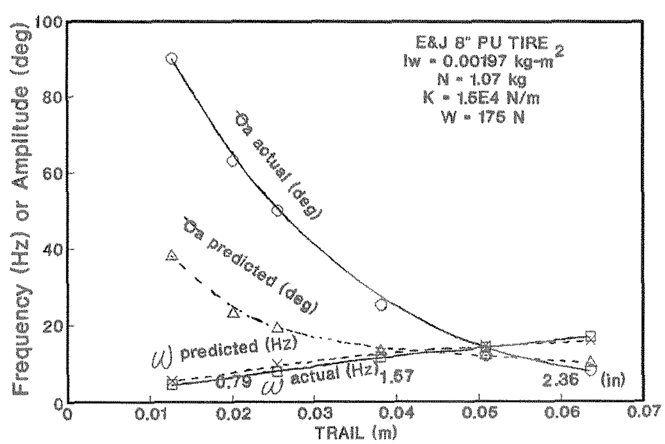

Figure 5.
Shimmy frequency and amplitude versus trail. I_w, mass moment of inertia of wheel about a wheel diameter; N, mass of caster unit; K, spring constant; W, load; θa, angular amplitude; ω, frequency.

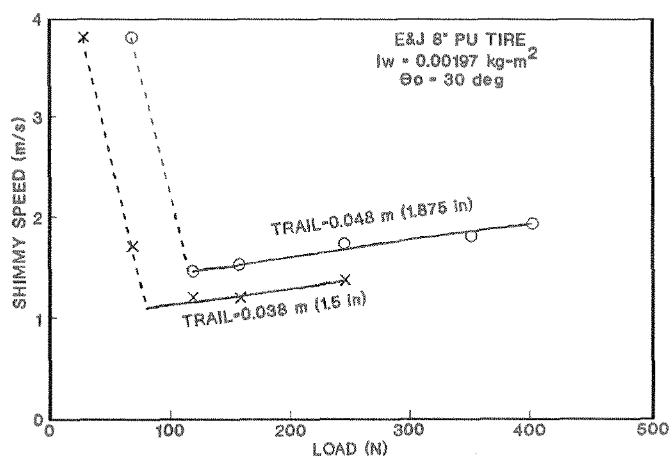

Figure 6.
Low load stability. I_w, mass moment of inertia of wheel about a wheel diameter; θ0, initial angular displacement.

Damping by Tire/Floor Friction

Experiments show that the sealed ball bearings of the caster spindle can only exert a frictional moment for two bearings on the order of 2×0.005=0.01 N-m, whereas a calculation of M^f using Equation 4 and the data in **Figure 4** gives a value for frictional moment of 0.9 N-m of which the spindle bearing frictional moment is only 1.1 percent. From **Figure 6,** it is shown that changing the load on the caster causes M_f to change essentially linearly with loads above 100 N.

One of the basic assumptions for caster wheel shimmy of aircraft (7) and wheelchairs is that there is no skidding of the wheel in a normal direction as it corners or oscillates. A simple test of this concept was demonstrated by pushing a loaded wheelchair along a floor covered with flour (8). As best that could be determined the shimmy motion of the caster, for the wheelchair speeds used, did not show any gross skidding of the wheel; the wheel tracks the motion without gross skidding. In this test, both the wheelchair frame and the caster oscillated, facts that must be considered in any conclusions.

Figure 7 is based on experimental observation of auto tire contact on a glass surface (9). These studies show that the tire tread deformation is parallel to the direction of travel, which supports the no-slip observations. More importantly, there is a region behind the adhesion area where the deformation exceeds the available tire-floor friction force and part of the tire contact goes into an angular sliding mode. In the case of wheelchair caster wheels (and aircraft), it is concluded that the wheel does not show gross skidding during shimmy. However, there is a torsional moment on the caster, due to the angular deflection of the tire about a vertical line through the tire-floor contact, where frictional energy is being dissipated.

A plot of wheelchair tire torsional friction from (4, **Figure 7**) for a tile floor, using data for M^f derived from data presented in **Figures 4 and 6** and calculated using Equation 4, is shown in **Figure 8**. A study of 13 different wheelchair caster tires by Frank and Abel (10) gave torsional friction data falling within the scatter of the data shown on **Figure 8**. There was no specific trend for "soft" or "hard" tire frictional torque. These results suggest the need for a new tire with a higher torsional friction for the given caster load, but without degrading rolling resistance. So far, this does not seem to be an available solution for suppressing shimmy.

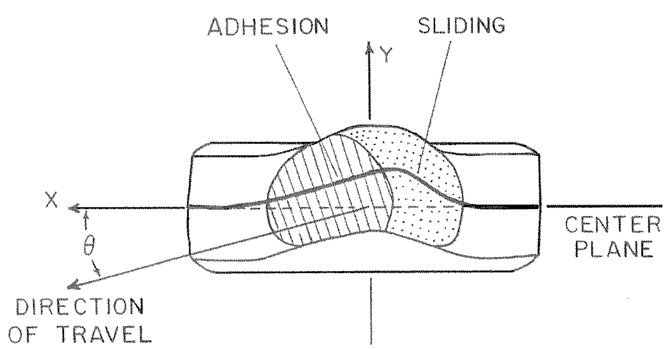

Figure 7.
Cornering tire deformation, exaggerated. θ, angular displacement.

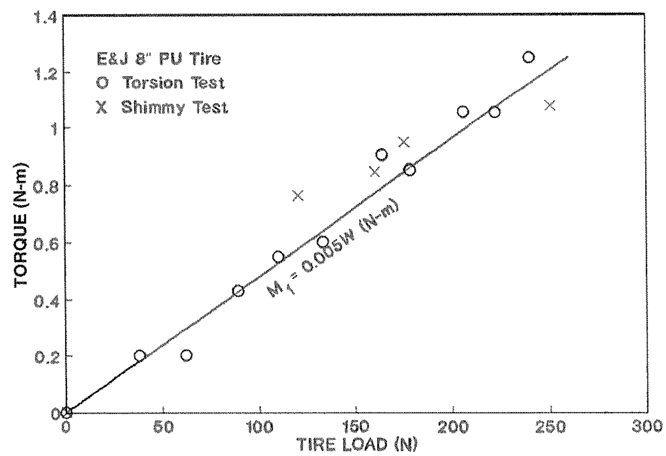

Figure 8.
Tire torque on tile. M_f=coulomb friction torque; W=load on caster.

Frictional Damping at the Spindle

In all but the cheapest wheelchairs, the design of the casters makes use of a sliding frictional damper in the spindle support to improve the shimmy characteristics. We will use this concept, in the form of a dry friction bearing at the top of the spindle, to demonstrate the theory.

Figure 9 is a suggested design, using a ball bearing for the load- bearing, lower spindle bearing, but replacing the upper ball bearing with a dry friction Delrin sleeve bearing having a coefficient of friction of 0.35. It is assumed that the distance between bearings is the same as the trail. With the load on one caster of 175 N (39 lbf), and a spindle shaft radius of 0.00635 m (0.25 in), the frictional moment can be determined for the tire, using **Figure 8**, to get 0.875 N-m; the Delrin bearing gives a frictional moment of 0.389 N-m. To apply Equation 4 to this design let M_f=0.875+0.389=1.264 N-m. The result is an 18.9 percent increase in the critical shimmy velocity from use of the Delrin bearing over use of the spindle supported by two ball bearings.

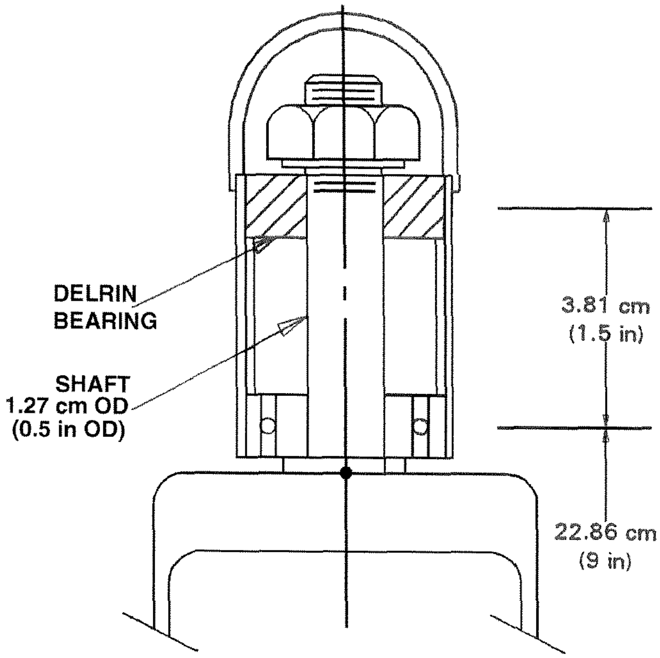

Figure 9.
Friction bearing design.

Hydraulic Damping

We will next consider the theory applied to a commercial, hydraulic damper attached to the spindle of a caster (6).

The hydraulic damper was a commercial Kinetrol, Model KD-A1-DD, and was set to a damping constant of 0.11 N-m-s/rad. For the hydraulic damper tests, M_f=0.46 N-m, I_w=0.0018 kg-m^2 (N-m-s^2), T=0.0254 m, and θ_0=0.349 rad. In this example, there are two damping coefficients; one is the frictional damping at the tire contact, and the second is the damping due to the hydraulic damper. Applying Equation 1 to these test data, gives

$$\frac{4M_f}{\pi\theta_0}\frac{T}{V_c}+C_D=I_w\frac{V_c}{T} \qquad [5]$$

The solution to quadratic Equation 5 is

$$\frac{V_c}{T}=\frac{C_D}{2I_w}\pm\sqrt{\left(\frac{C_D}{2I_w}\right)^2+\frac{4M_f}{\pi\theta_0 I_w}} \qquad [6]$$

Solving Equation 6 for the data listed above Equation 5 gives V_c=1.8 m/s, and without the hydraulic damping gives V_c=0.78 m/s, resulting in a critical shimmy wheelchair velocity ratio of 2.3, which is in agreement with the test data presented by Mochel (6).

Hydraulic dampers are expensive but they have the advantage that damping is linearly proportional to the *angular velocity* of the caster rather than *angular displacement,* as in the case of frictional damping. Thus, the action of turning a wheelchair at slow speeds is not penalized by a hydraulic damper, whereas for frictional dampers the turning torque is the same at all turning speeds.

Ball Bearing Torque

A new theory of hydraulic damping is presented next, for damping that can be developed by the ball bearings with a high viscosity oil or grease. For the usual light oil-filled ball bearings, the drag due to the oil is so small it is negligible.

The viscous frictional torque of rolling element bearings was developed empirically, and the basic equations are reviewed by Harris (11). Neglecting seal friction torque, the torque due to viscous churning of the lubricant in the bearing, M_ν, can be written as follows:

$$M_\nu=4.4x10^{-10}f_0d_m^3(\nu\dot{\theta})^{2/3} \qquad [7]$$

where the bearing viscous friction factor, f_o, is 5 for grease-lubricated, deep groove ball bearings (11, p. 429), d_m=19.5 mm for typical wheelchair casters, the kinematic viscosity, ν, equals 2000 centistokes (cSt) for grease or heavy oils (about the maximum commercially available), and $\dot{\theta}$ is the angular velocity in rad/s.

First, it is necessary to develop an expression for the equivalent hydraulic damping coefficient by setting the work per cycle for a simple hydraulic damper equal to the work per cycle using Equation 7, and assuming sinusoidal motion; the method is shown in (4, Appendix C).

The work per cycle due to Equation 7 is:

$$WORK(\nu)=\int M_\nu d\theta=\int K_\nu\dot{\theta}^{2/3}d\theta \qquad [8]$$

Expanding Equation 8 in terms of sinωt (12, p. 70), results in

$$\begin{aligned}WORK(\nu)&=\int K_\nu(\omega\theta_a sin\omega t)^{2/3}\theta_a\sin(\omega t)d(\omega t)\\&=4K_\nu\omega^{2/3}\theta_a^{5/3}\int_0^{\pi/2}\sin^{5/3}(\omega t)d(\omega t)\end{aligned} \qquad [9]$$

The integral is a Beta function, and can be evaluated from a table of integrals by Gradshteyn and Ryzhik (13, pp. 369, 950, and 938) to give a value of 0.84131 (or one can use a mathematics computer program). Substituting for the integral in Equation 9 gives

$$WORK(v)=3.3652K_v\omega^{2/3}\theta_a^{5/3}$$
$$K_v=4.4x10^{-10}f_0d_m^3v^{2/3} \qquad [10]$$

The work per cycle for a hydraulic damper, from (4, Equation 2C) is WORK=$C_D\pi\omega\theta_a^2$, and setting this work equal to that of Equation 10, gives Equation 11 (where Equation 3 was used to simplify Equation 11),

$$C_D=\frac{3.3652K_v}{\pi(\omega\theta_a)^{1/3}}$$
$$=\frac{3.3652K_v}{\pi\theta_0^{1/3}}\left(\frac{T}{V_c}\right)^{1/3} \qquad [11]$$

Applying Equation 1 with a frictional damper for the tire contact (Equation 4), and the ball bearing torque damping coefficient from Equation 11, the shimmy stability equation becomes:

$$\left(\frac{V_c}{T}\right)^2-\frac{3.3652K_v}{\pi\theta_0^{1/3}I_w}\left(\frac{V_c}{T}\right)^{2/3}-\frac{4M_f}{\pi I_w\theta_0}=0 \qquad [12]$$

Substituting f_0=2 (bearings)×5=10 (from Harris (11)), d_m=19.5 mm (typical wheelchair caster bearings), v=2000 cSt (maximum value for grease), I_w=0.0018 kg-m² (N-m-s²), M_f=0.6 N-m at 120 N (from **Figure 8**), θ_0=20º=0.34907 rad (usually about 10º but used 20º for design), and T=1.5 in=0.0381 m, one gets:

$$\left(\frac{V_c}{T}\right)^2-4.2878\left(\frac{V_c}{T}\right)^{2/3}-1215.8=0 \qquad [13]$$

The solution to Equation 13 is V_c/T=35.53 s^{-1}. If the grease-filled bearings are replaced by light oil-filled bearings, the solution is simply the square root of the absolute value of the third term on the left, resulting in V_c/T=34.869 s^{-1}. These calculations predict a 1.9 percent increase in V_c by using the grease-filled ball bearings. Here we see the value of the theory in design. For a trial-and-error design using grease-filled ball bearings, the theory shows that for typical mineral oil greases there is no advantage for the concept.

New Design

Although mineral oil greases used in the spindle ball bearings do not offer significant shimmy damping, a synthetic silicone lubricant is available in much higher viscosity. The effect of changing the lubricant viscosity to a much higher value was therefore considered. Choosing a silicone fluid of v=500,000 cSt, Equation 13 is changed as follows:

$$\left(\frac{V_c}{T}\right)^2-170.16\left(\frac{V_c}{T}\right)^{2/3}-1215.8=0 \qquad [14]$$

The solution to Equation 14 is V_c/T=62.4 s^{-1}, a predicted 79 percent increase in the critical shimmy velocity over the case when the bearings are filled with light oil.

Silicone fluids are especially useful for linear and torsional dampers, viscous drive clutches, dashpots, and as the base fluid for greases used in these devices (14, 15). The available viscosity of silicones varies from 5–1,000,000 cSt at 25ºC, and their outstanding properties include the least change in viscosity with temperature of all synthetics, good thermal stability, good oxidation stability, low surface tension, and non-corrosive characteristics. However, Bryson (14) points out that the most common silicone, dimethyl, has poor steel-on-steel lubricity. Lubricity can be defined as the effect of reducing sliding friction due to the lubricant. A methyl alkyl silicone overcomes the latter shortcoming, but is more expensive. In addition, silicones are shear thinning (16, 17, and 15). Shear thinning lubricants show a much-decreased viscosity proportional to bearing shear. At low shear, typical of caster shimmy, this may not be a problem.

DISCUSSION

In the original paper on wheelchair caster shimmy there were some errors concerning the discussion of the results of the theory, but the basic theory calculations are valid. At the bottom of (4, p. 21) it is stated incorrectly that the spindle bearing friction has increased with trail. It is now understood that the damping friction between the tire and floor is the main damping input, and it is relatively constant if the load on the caster is constant. An important variable that was not controlled well early in testing is the initial displacement of the caster. If the tests used a constant initial displacement and caster load, the plot of critical velocity versus trail would be a straight line.

Given the constraints on the design of wheelchair casters, this study shows that shimmy prevention, under all conditions, depends on having sufficient damping. A new concept in damper design, where a very viscous lubricant is used in the wheelchair caster spindle ball bearings, is predicted to offer a significant increase in onset shimmy velocity.

CONCLUSIONS

The original shimmy theory presented by the authors in 1984 has been reexamined, and found satisfactory for predicting the performance of typical wheelchair casters. There are, however, some regions of operation where the assumptions used in the analysis break down. The theory is inaccurate for casters under a low load, such as when the wheelchair is empty or the user shifts the load off the casters. However, at low loads the casters are found to be more shimmy stable than predicted by this theory. In addition, the theory is inaccurate for casters with small trail (e.g., less than 0.04 m (1.57 in)).

Understanding the theory of damping for the caster is the major thrust of this paper. This paper presents information on damping of spindle bearings, tire/floor friction, add-on frictional dampers, add-on hydraulic dampers, and a new theory of ball bearing torque damping. The latter method of damping has the theoretical possibility of improving shimmy prevention by more than 79 percent, and is being considered for further research and patentability.

APPENDIX

There were a number of typographical errors in the 1984 paper on shimmy (4) that makes the analysis difficult to follow.

The corrections are:

1. Equation 4: 1/t should be 1/T

2. Equation 5B: the 4th term on left is negative

3. Equation 26B: 2nd term on right, 1/t should be 1/T

4. Below Equation 8C: >> should be <<, and

5. In Nomenclature list: $N=M_F+{}_1$,kg.

GLOSSARY

a, b. Dimensions, m
C_D. Damping coefficient, N-m/rad/s
d_m. Bearing pitch diameter, mm
F_b. Force on bearing, N
f_o. Bearing viscous friction factor
I_w. Mass moment of inertia of wheel about a wheel diameter, kg-m_2
K. Spring constant, N/m
K_ν. Viscous friction torque factor, Equation 10
M_F. Mass of fork, kg
M_1. Mass of spindle bearing housing, kg
M_f. Coulomb friction torque, N-m
M_ν. Bearing viscous friction torque, N-m
N. Mass of caster unit, kg
T. Caster trail, m
V. Wheelchair velocity, m/s
V_c. Shimmy critical velocity, m/s
W. Load, N
x,y,z. Coordinates, m
θ. Angular displacement, rad
θ_0. Initial angular displacement, rad
$\dot{\theta}$. Angular velocity, rad/s
θ_a. Angular amplitude, rad
ν. Kinematic viscosity, cSt
ω. Frequency, rad/s
ω_c. Shimmy critical frequency, rad/s

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