Appendix Equations

The three monotonic functions were—

\[ Y = \{a\} \times (1 - \text{Exp}[-((X + 2)/\{c\})^{\{b\}}])] \]  \hspace{1cm} \text{[Weibull]} \hspace{1cm} \text{Equation 1(a)}

\[ Y = \{a\}/(1 + \text{Exp}[-\{b\} \times (X - \{c\})])] \]  \hspace{1cm} \text{[Logistic]} \hspace{1cm} \text{Equation 1(b)}

\[ Y = \{a\} \times \text{Exp}[-\text{Exp}[-\{b\} \times (X - \{c\})])] \]  \hspace{1cm} \text{[Gompertz]} \hspace{1cm} \text{Equation 1(c)}

Initial \{a\} for regression was set equal to \(Y(\text{max})\).

Initial \{b\} for regression was set as indicated in Equations 6(a)–(c) using the line slope \((\Delta Y/\Delta X, \text{Equation 3})\) from point of smallest ordinate with largest abscissa \((Y(\text{min}))\) to point of largest ordinate with smallest abscissa \((Y(\text{max}))\).

Initial \{c\} for regression was set equal to the abscissa midpoint of \(\Delta Y/\Delta X\).

\[ \frac{dY}{dX}_{\text{int}} = 0.37 \times \{a\} \times \{b\}/\{c\} \]  \hspace{1cm} \text{[Weibull]} \hspace{1cm} \text{Equation 2(a)}

\[ \frac{dY}{dX}_{\text{int}} = \{a\} \times \{b\}/\{c\} \times ((\{b\} - 1)/\{b\})^{(\{b\} - 1)/\{b\}} \times \text{Exp}[-(\{b\} - 1)/\{b\}] \]  \hspace{1cm} \text{Equation 2(b)}

\[ \frac{dY}{dX}_{\text{int}} = \frac{dY}{dX}_{\text{int}} = 0.25 \times \{a\} \times \{b\} \]  \hspace{1cm} \text{[Logistic]} \hspace{1cm} \text{Equation 2(c)}

\[ \frac{dY}{dX}_{\text{int}} = \frac{dY}{dX}_{\text{int}} = 0.37 \times \{a\} \times \{b\} \]  \hspace{1cm} \text{[Gompertz]} \hspace{1cm} \text{Equation 2(d)}

\[ \Delta Y/\Delta X = (Y(\text{max}) - Y(\text{min}))/(X(\text{max}) - X(\text{min})) = 115 \text{ wpm/logMAR} \]  \hspace{1cm} \text{Equation 3}

Evaluated for the typical observer (see Figure 2(a) main text):

Initial \{a\} = \(Y(\text{max}) = 149 \text{ wpm} \)  \hspace{1cm} \text{Equation 4}

Initial \{c\} = \((X(\text{max}) - X(\text{min}))/2 + X(\text{min}) = 2.55 \text{ logMAR} \)  \hspace{1cm} \text{Equation 5}

Initial \{b\} = \(\Delta Y/\Delta X \times (\text{init}\{c\}/\text{init}\{a\}) \times (1/0.37) = 5.32 \)  \hspace{1cm} \text{[Weibull]} \hspace{1cm} \text{Equation 6(a)}

Initial \{b\} = \(\Delta Y/\Delta X \times (1/\text{init}\{a\}) \times (1/0.25) = 3.09 \)  \hspace{1cm} \text{[Logistic]} \hspace{1cm} \text{Equation 6(b)}

Initial \{b\} = \(\Delta Y/\Delta X \times (1/\text{init}\{a\}) \times (1/0.37) = 2.09 \)  \hspace{1cm} \text{[Gompertz]} \hspace{1cm} \text{Equation 6(c)}

Inflexion coordinates for the monotonic Weibull (Equation (1a)) were—
\[ X_{i} = \{c\} \times (1 - 1/\{b\})^{1/\{b\}} - 2 \quad \text{Equation 7} \]

\[ Y_{i} = \{a\} \times (1 - \text{Exp}[(1 - \{b\})/\{b\}]) \quad \text{Equation 8} \]

Nonmonotonic Weibull models:

\[ Y = \{a\} \times (2 - \text{Exp}[-((X + 2)/\{c\})^{\{b\}}] - \text{Exp}[-((X + 1)/\{g\})^{\{e\}}]) \quad \text{Equation 9(a)} \]

\[ Y = \{a\} \times (-\text{Exp}[-((X + 2)/\{c\})^{\{b\}}] + \text{Exp}[-((X + 1)/\{g\})^{\{e\}}]) \quad \text{Equation 9(b)} \]

Rearranging each:

\[ Y = \{a\} \times (1 - \text{Exp}[-((X + 2)/\{c\})^{\{b\}}] +/\- \{a\} \times (1 - \text{Exp}[-((X + 1)/\{g\})^{\{e\}}])) \quad \text{Equation 9(c)} \]

Initial \(\{a\}, \{b\},\) and \(\{c\}\) were set to values given by monotonic regression \textit{Equation 1(a)}. Initial \(\{g\}\) was set to the value of initial \(\{c\}\). Initial \(\{e\}\) was set to 10.