Appendix Equations

The three monotonic functions were—

$Y = \{a\} \times (1 - \mathrm{Exp}[-((X + 2)/\{c\})^{\{b\}}])$ [Weibull] **Equation 1(a)**

$Y = \{a\}/(1 + \mathrm{Exp}[-\{b\} \times (X - \{c\})])$ [Logistic] **Equation 1(b)**

$Y = \{a\} \times \mathrm{Exp}[-\mathrm{Exp}[-\{b\} \times (X - \{c\})]]$ [Gompertz] **Equation 1(c)**

Initial$\{a\}$ for regression was set equal to Y(max).

Initial $\{b\}$ for regression was set as indicated in **Equations 6(a)–(c)** using the line slope ($\Delta Y/\Delta X$, **Equation 3**) from point of smallest ordinate with largest abscissa (Y(min)) to point of largest ordinate with smallest abscissa (Y(max).

Initial $\{c\}$ for regression was set equal to the abscissa midpoint of $\Delta Y/\Delta X$.

$dY/dX_{\{c\}-2} = 0.37 \times \{a\} \times \{b\}/\{c\}$ [Weibull] **Equation 2(a)**

$dY/dX_{\{i\}} = \{a\} \times \{b\}/\{c\} \times (\{b\} - 1)/\{b\})^{(\{b\}-1)/\{b\}} \times \mathrm{Exp}[-(\{b\} - 1)/\{b\}]$ **Equation 2(b)**

$dY/dX_{\{c\}} = dY/dX_i = 0.25 \times \{a\} \times \{b\}$ [Logistic] **Equation 2(c)**

$dY/dX_{\{c\}} = dY/dX_i = 0.37 \times \{a\} \times \{b\}$ [Gompertz] **Equation 2(d)**

$\Delta Y/\Delta X = (Y(\max) - Y(\min))/(X(\max) - X(\min)) = 115$ wpm/logMAR **Equation 3**

Evaluated for the typical observer (see **Figure 2(a)** main text):

Initial $\{a\} = Y(\max) = 149$ wpm **Equation 4**

Initial $\{c\} = (X(\max) - X(\min))/2 + X(\min) = 2.55$ logMAR **Equation 5**

Initial $\{b\} = \Delta Y/\Delta X \times (\mathrm{init}\{c\}/\mathrm{init}\{a\}) \times (1/0.37) = 5.32$ [Weibull] **Equation 6(a)**

Initial $\{b\} = \Delta Y/\Delta X \times (1/\mathrm{init}\{a\}) \times (1/0.25) = 3.09$ [Logistic] **Equation 6(b)**

Initial $\{b\} = \Delta Y/\Delta X \times (1/\mathrm{init}\{a\}) \times (1/0.37) = 2.09$ Gompertz] **Equation 6(c)**

Inflexion coordinates for the monotonic Weibull (**Equation (1a)**) were—

$X_i = \{c\} \times (1 - 1/\{b\})^{\mathbf{1/\{b\}}} - 2$ **Equation 7**

$Y_i = \{a\} \times (1 - \mathrm{Exp}[(1 - \{b\})/\{b\}])$ **Equation 8**

Nonmonotonic Weibull models:

$Y = \{a\} \times (2 - \mathrm{Exp}[-((X + 2)/\{c\})^{\mathbf{\{b\}}}] - \mathrm{Exp}[-((X + 1)/\{g\})^{\mathbf{\{e\}}}])$ **Equation 9(a)**

$Y = \{a\} \times (-\mathrm{Exp}[-((X + 2)/\{c\})^{\mathbf{\{b\}}}] + \mathrm{Exp}[-((X + 1)/\{g\})^{\mathbf{\{e\}}}])$ **Equation 9(b)**

Rearranging each:

$Y = \{a\} \times (1 - \mathrm{Exp}[-((X + 2)/\{c\})^{\mathbf{\{b\}}}] +/- \{a\} \times (1 - \mathrm{Exp}[-((X + 1)/\{g\})^{\mathbf{\{e\}}}]))$ **Equation 9(c)**

Initial $\{a\}$, $\{b\}$, and $\{c\}$ were set to values given by monotonic regression **Equation 1(a)**.

Initial $\{g\}$ was set to the value of initial $\{c\}$.

Initial $\{e\}$ was set to 10.