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APPENDIX 1

This appendix presents the mechanical equations used for elaborating the mechanical model expressed in **Equation (1)**, which represents the COM deceleration of a loaded MWC rolling straight forward on a horizontal floor without any propulsive force. Under this condition, the only forces applied to the loaded MWC are the total weight ($\vec{W}$) and the ground reaction forces on the front ($\vec{R}_f$) and rear wheels ($\vec{R}_r$), which make it possible to write the following equations along the fore-aft and vertical directions of a Galilean reference frame:

$$R_{fx} + R_{rx} = m\gamma_G \quad \text{[Eq. 1A]}$$

$$W + R_{fN} + R_{rN} = 0 \quad \text{[Eq. 2A]}$$

where R_{fx} and R_{rx} are the fore-aft components and R_{fN} and R_{rN} are the normal components of the ground reaction forces applied on the front and rear wheels, respectively; m is the total mass; and γ_G is the fore-aft COM deceleration of the loaded MWC.

When focusing on the front and rear wheels, the net moment (sum of external moments) of each wheel acting at its center is equal to the time differentiation of the angular momentum. Assuming that the MWC is rolling straight forward without slipping on the ground, the latter can be expressed from the fore-aft COM deceleration. Thus, in the transverse direction:

$$r_f\, R_{fx} + \lambda_f\, R_{fN} = -I_f \frac{\gamma_G}{r_f} \quad \text{[Eq. 3A]}$$

$$r_r R_{rx} + \lambda_r R_{rN} = -I_r \frac{\gamma_G}{r_r} \quad \text{[Eq. 4A]}$$

where λ_f and λ_r are the front and rear wheels' *RP;* r_f and r_r are the front and rear wheels' radii; and I_f and I_r are the moments of inertia of the two front wheels and the two rear wheels around their rotational axes, respectively.

Therefore, R_{fx} and R_{rx} can be expressed by **Equations (5A)** and **(6A)**:

$$R_{fx} = -\frac{\lambda_f}{r_f} R_{fN} - \frac{I_f}{{r_f}^2} \gamma_G \quad \text{[Eq. 5A]}$$

$$R_{rx} = -\frac{\lambda_r}{r_r} R_{rN} - \frac{I_r}{{r_r}^2} \gamma_G \quad \text{[Eq. 6A]}$$

Replacing the last two equations (**(5A)** and **(6A)**) in **Equation (1A)** and gathering the terms in γ_G makes it possible to write the relation between the normal ground reaction forces and COM deceleration of the loaded MWC:

$$-\frac{\lambda_f}{r_f} R_{fN} - \frac{\lambda_r}{r_r} R_{rN} = \left(m + \frac{I_f}{{r_f}^2} + \frac{I_r}{{r_r}^2} \right) \gamma_G \quad \text{[Eq. 57]}$$

R_{rN} can easily be extracted from equation 2A and then replaced in **Equation (7A)** in order to write the following expression of R_{fN}:

$$R_{fN} = -\frac{\lambda_r}{r_r} \left(\frac{r_f \, r_r}{\lambda_r r_f - \lambda_f r_r} \right) W + \ldots \left(m + \frac{I_f}{{r_f}^2} + \frac{I_r}{{r_r}^2} \right) \left(\frac{r_f \, r_r}{\lambda_r r_f - \lambda_f r_r} \right) \gamma_G \quad \text{[Eq. 8A]}$$

On the other hand, the net moment (sum of external moments) acting on the loaded MWC can be expressed at the COM and is equal to the time differentiation of the angular momentum, which is drastically simplified when the MWC is loaded with unmovable masses. This relation can thus be expressed in the transverse direction as follows:

$$\left(d_f+\lambda_f\right)R_{fN}+\left(-d_r+\lambda_r\right)R_{rN}+\ldots \qquad \text{[Eq. 9A]}$$
$$h\left(R_{fx}+R_{rx}\right)=-\left(\frac{I_1}{r_1}+\frac{I_2}{r_2}\right)\gamma_G$$

where d_f and d_r are the horizontal distances between the COM and the front and rear wheels hubs, respectively; d_f+d_r is the wheelbase (w_b); and h is the height of the COM with respect to the ground.

Using **Equations (1A)** and (**2A)** in **Equation (9A)** gives:

$$\left(m+\frac{I_f}{r_f h}+\frac{I_r}{r_r h}\right)h\gamma_G=-\left(w_b+\lambda_f-\lambda_r\right)R_{fN}+\ldots \qquad \text{[Eq. 10A]}$$
$$\left(-d_r+\lambda_f\right)W$$

The expression for R_{fN} (**Eq. (6A)**) can be replaced in **Equation (8A)**, which becomes:

$$\left(\left(w_b+\lambda_f-\lambda_r\right)\left(m+\frac{I_f}{{r_f}^2}+\frac{I_r}{{r_r}^2}\right)\left(\frac{r_f\, r_r}{\lambda_r r_f-\lambda_f r_r}\right)+\ldots\right.$$
$$\left.\left(m+\frac{I_f}{r_f h}+\frac{I_r}{r_r h}\right)h\right)\gamma_G=$$
$$\left(\left(w_b+\lambda_f-\lambda_r\right)\frac{\lambda_r}{r_r}\left(\frac{r_f\, r_r}{\lambda_r r_f-\lambda_f r_r}\right)-\left(d_r-\lambda_r\right)\right)W \qquad \text{[Eq. 11A]}$$

Then, multiplying the previous expression by ($\lambda_r r_f-\lambda_f r_r$) and dividing by $r_f r_r w_b$ gives:

$$\left(\left(m+\frac{I_f}{{r_f}^2}+\frac{I_r}{{r_r}^2}\right)\left(1+\frac{\lambda_f-\lambda_r}{w_b}\right)+\ldots\right.$$
$$\left.\left(m+\frac{I_f}{r_f h}+\frac{I_r}{r_r h}\right)\left(\frac{\lambda_r}{r_r}-\frac{\lambda_f}{r_f}\right)\frac{h}{w_b}\right)\gamma_G=$$
$$\left(\frac{\lambda_f}{r_f}\frac{d_r}{w_b}+\frac{\lambda_r}{r_r}\frac{d_f}{w_b}+\frac{\lambda_f\lambda_r}{r_f r_r}\frac{r_f-r_r}{w_b}\right)W \qquad \text{[Eq. 12A]}$$

As $W=-mg$, this equation makes it possible to express **Equation (1)**.

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APPENDIX 2

Two simplified forms (**Equations (1B)** and **(2B)**) of **Equation (1)** were evaluated by means of 100,000 randomized computations of **Equations (1)**, **(1B)**, and **(2B)** using specific values either reported or measured on MWCs: the *RP* values ranged from 1 to 3 mm [1–4]; the front wheel radii ranged from 30 to 100 mm and their moments of inertia ranged from 0.005 to 0.002 kg.m^2; the rear wheel radii ranged from 260 to 330 mm and their moments of inertia ranged from 0.1 to 0.2 kg.m^2; the wheelbase ranged from 300 to 450 mm; the COM height of the loaded MWC ranged from 500 to 700 mm; the total mass ranged from 75 to 100 kg [5]; and the fore-aft mass distribution ranged from 30 to 60 percent of the mass distributed on the front wheels [3].

$$\gamma_G = -mg \frac{\left(\frac{\lambda_f}{r_f} \frac{d_r}{w_b} + \frac{\lambda_r}{r_r} \frac{d_f}{w_b} \right)}{\left(m + \frac{I_f}{r_f^2} + \frac{I_r}{r_r^2} \right)} \quad \text{[Eq. 1B]}$$

$$\gamma_G = -g \left(\frac{\lambda_f}{r_f} \frac{d_r}{w_b} + \frac{\lambda_r}{r_r} \frac{d_f}{w_b} \right) \quad \text{[Eq. 2B]}$$

The results of these simulations showed errors with respect to **Equation (1)**, which remained at around 3.5 percent when using **Equation (1B)** and around 3.3 percent with **Equation (2B)**.

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