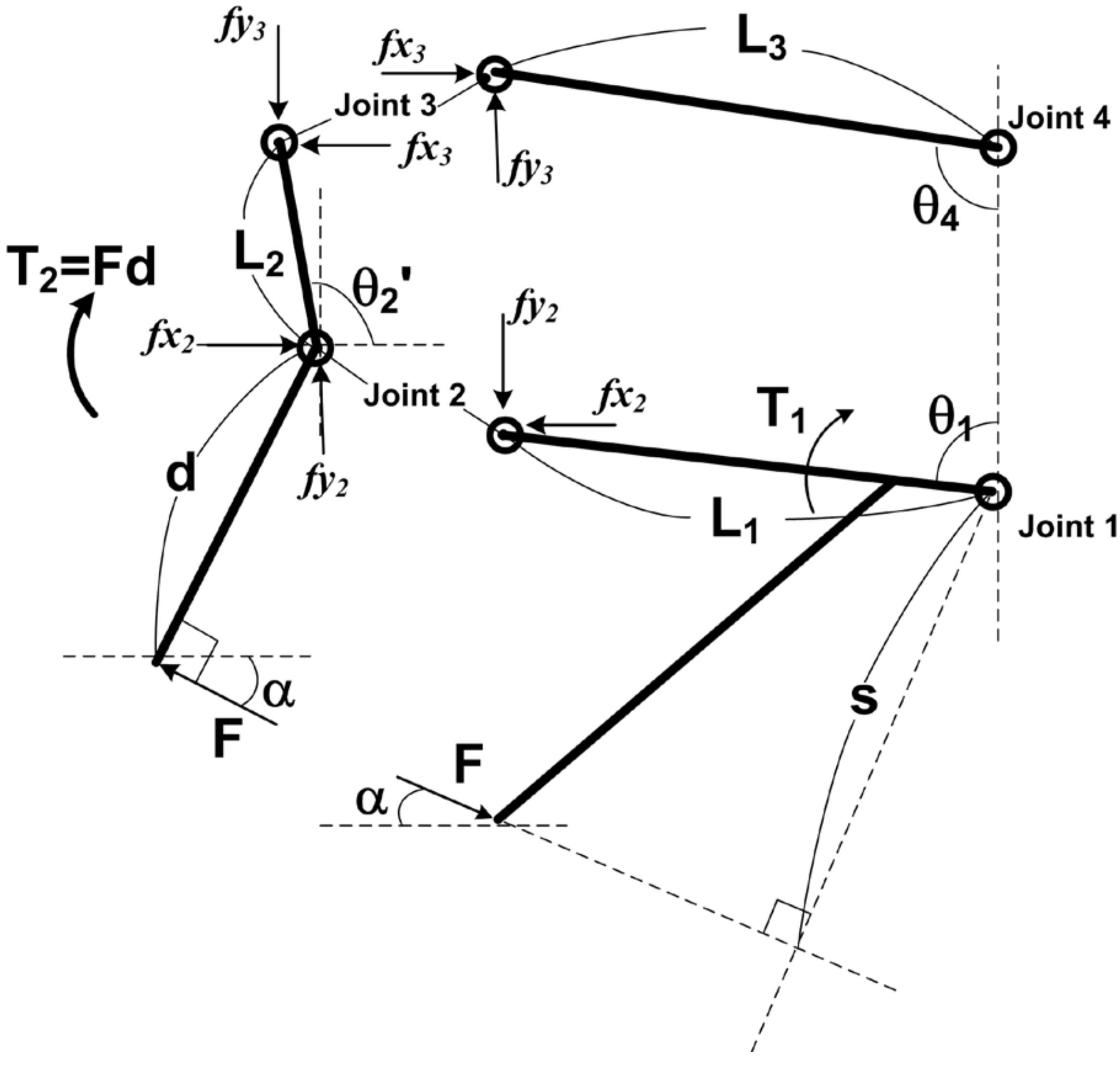
APPENDIX: Static Analysis of WDFHO

When the wrist extension torque (T_1) is applied to link 1 (L_1), the pinch force (F) on link 1 (L_1) and the operating lever (L_2) constructs force equilibrium. The free body diagrams of each link at the equilibrium are drawn as **Figure A**. At the static equilibrium, all forces and moments at each link do not create movement; therefore, sums of the forces and along x-axis and y-axis as well as the sum of the moments around a joint should be zero.

(a)



(b)

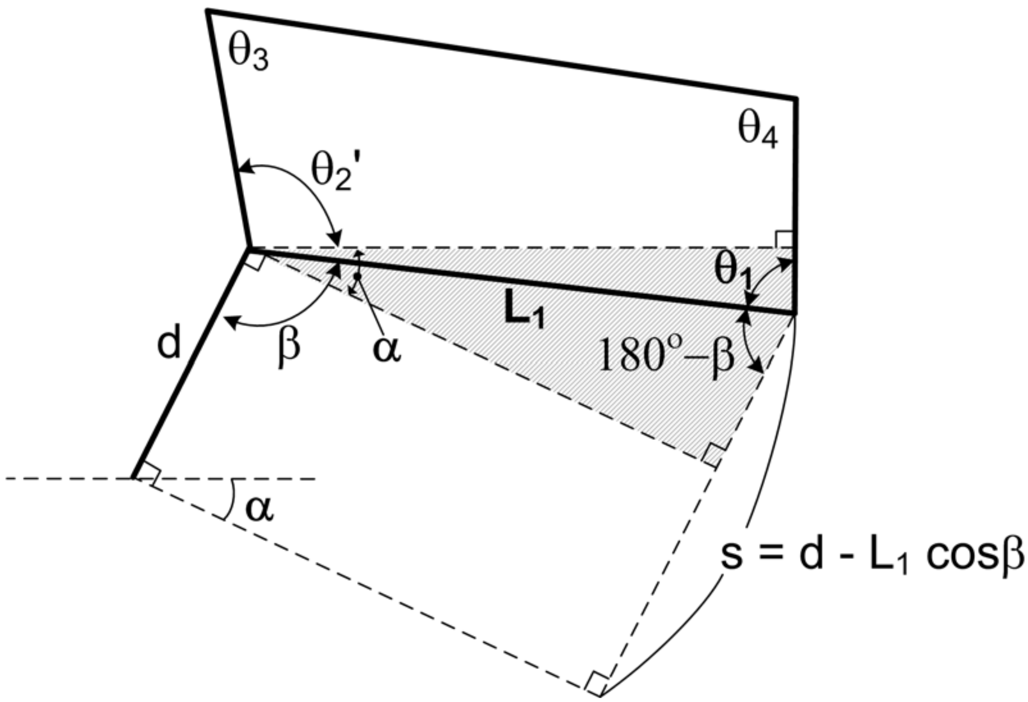


Figure A. Schematic diagrams for static force analysis in the example of right hand side: **(a)** free body diagrams of link 1, 2, and 3, **(b)** geometry of the WDFHO. T_1 = wrist extension torque, T_2 = MCP joint torque, F = three-point pinch force, d = length from the radial side of the second MCP joint to the fingertip, L_1 = length from the radial side of the second MCP joint to the distal tip of the radial styloid, L_2 = length of operating lever, L_3 = length of actuating rod, $\theta_1 \sim \theta_4$ = angles between four linkages, θ_2' = angle of operating lever from horizontal line, Joint 1 ~ 4 = joints between four linkages, s = moment arm of F acting on joint 1, fx_i = horizontal force at joint *i*, fy_i = vertical force at joint *i*.

For link 1, the moment equilibrium around joint 1 is derived as

$$T_1 - F \cdot s - L_1 \cos\theta_1 fx_2 - L_1 \sin\theta_1 fy_2 = 0\,. \qquad \textbf{(A1)}$$

For link 2, force equations along x and y axis are

$$fx_3 - fx_2 + F\cos\alpha = 0, \quad \textbf{(A2)}$$

and $$fy_2 - fy_3 + F\sin\alpha = 0. \quad \textbf{(A3)}$$

The moment equation at link 2 around joint 2 is constructed as

$$F \cdot d - L_2 \sin\theta_2' fx_3 + L_2 \cos\theta_2' fy_3 = 0. \quad \textbf{(A4)}$$

At link 3, the sum of moment around joint 4 is expressed as the following:

$$L_3 \sin\theta_4 fy_3 - L_3 \cos\theta_4 fx_3 = 0. \quad \textbf{(A5)}$$

Since $T_2 = F \cdot d$, solving **Equations A1 ~ A5** for T_1 and T_2, and using trigonometric identities, the following equation is obtained:

$$\frac{T_1}{T_2} = \frac{s}{d} + \frac{L_1 \sin\left(\theta_1 + \theta_4\right)}{L_2 \cos\left(90^\circ - \theta_3\right)} + \frac{L_1}{d}\cos\left(\theta_1 + \alpha\right). \quad \textbf{(A6)}$$

Equation A6 can be further simplified from the geometry of the linkage (**Figure A(b)**).

By defining the angle between linkages L_1 and d as β, the moment arm of F acting on joint 1 (represented as in **Figure A**) can be expressed as

$$s = d - L_1 \cos\beta. \quad \textbf{(A7)}$$

Since the sum of four angles of the shaded rectangle (**Figure A (b)**) is 360 degrees,

$$\beta = \alpha + \theta_1. \quad \textbf{(A8)}$$

By replacing **Equation A7** and **A8** into **A6**, **Equation A6** is simplified as

$$\frac{T_1}{T_2} = 1 + \frac{L_1 \sin\left(\theta_1 + \theta_4\right)}{L_2 \cos\left(90^\circ - \theta_3\right)}$$

or

$$T_2 = \frac{L_2 \cos(90^\circ - \theta_3)}{L_2 \cos(90^\circ - \theta_3) + L_1 \sin(\theta_1 + \theta_4)} T_1 . \quad \textbf{(A9)}$$

Since T_2 is equivalent to the pinch force (F) multiplied by the moment arm (d), the resultant pinch force is calculated as the following:

$$F = \frac{L_2 \cos(90^\circ - \theta_3)}{L_2 \cos(90^\circ - \theta_3) + L_1 \sin(\theta_1 + \theta_4)} \cdot \frac{T_1}{d} \quad . \qquad \textbf{(A10)}$$