COMPUTER OPTIMIZATION OF POLYCENTRIC PROSTHETIC KNEE MECHANISMS

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ABSTRACT

The design of mechanisms which approximate desired centrodes is discussed in this paper. The application to two prosthetic knee mechanisms is presented.

INTRODUCTION

The knee joint in a standard artificial limb is traditionally of the single-axis type which in the past has provided an acceptable function for many amputees. In this design, knee stability during weight-bearing is achieved by positioning the knee axis in such a way relative to the body-weight action line that the knee is extended. In addition, a moment from active hip extension muscles is required during the weight-bearing phase of the walking cycle. This means that the amputee must walk during weight-bearing over a fully extended (straight) knee, which is physiologically abnormal and contributes to the unnatural appearing gait of the above-knee (thigh) amputee. Furthermore, when amputations are performed through the knee joint, the resulting long stump leaves insufficient space for the single-axis knee mechanism. In this case, single-axis side joints are required, resulting in greater fabrication time and an unpleasant appearance of the finished appliance due to excessive width of the knee.

These and other shortcomings of the single-axis design have encouraged designers to seek other mechanisms for knee devices. The four-bar linkage, which yields polycentric (or many centered) action of the center of knee rotation, was decided upon in this case and the associated criteria established.
At the University of California Biomechanics Laboratory, an early optimization attempt was made using an adjustable model to find a four-bar-linkage configuration. Due to the number of parameters involved, this approach was laborious and did not yield an optimum.

For this paper, an optimizing technique employing a high speed digital computer was used. The mathematical model of the linkage was established and a computer program was written to perform the computations. In this program, a “desired” mechanism output is specified and eight parameters of a “guessed” mechanism are systematically altered until some criterion function is minimized (or reduced to an acceptable value).

STABILITY OF PROSTHESES

Radcliffe (1) demonstrated that for a prosthetic knee to be stable during weight-bearing, the axis of the knee should be located behind the bad line from the greater trochanter of the femur to the point of load bearing at the ankle, the approximate coronal plane in which bodyweight is transferred to the limb. Furthermore, for the amputee to be able to control the knee—that is, for him to be able to land on a flexed knee and still be able to control knee flexion by applying an extension moment with his hip exterior muscles during stance phase—the knee axis should be located above the anatomical knee and posterior to the tKA line. A single-axis knee at this position is not cosmetically acceptable when the amputee is sitting. A device must therefore be designed for which the effective knee center is high and to the rear when the knee is fully extended, but has the appearance of the normal knee when flexed 90 deg.

A successful device for shifting the knee center has been the four-bar linkage (1,2). In this polycentric knee, the socket for the stump is mounted on the knee block, which serves as the coupler of the linkage. The frame is considered to be the lower portion of the limb, the shank; the effective knee axis is the instant center between the coupler and the frame.

It would be desirable to specify the motion of the effective knee center so that force transmission properties can be optimized. This can be done by specifying the desired fixed and moving centrodes or by specifying only the fixed centrode and the position of the instant center at several coupler angles (this implicitly defines the moving centrode).

ANALYSIS OF THE FOUR-BAR LINKAGE
WITH THE COUPLER AS INPUT

For analysis purposes the shank will be considered as being the frame
and the knee block as being the coupler. A reference coordinate system is drawn at any convenient point in the frame link (Fig. 1). The crank pins A and B on the knee block rotate about their crank centers \( O_A \) and \( O_B \) respectively. The link \( O_A A \) is taken to be link 2, the coupler \( AB \) is link 3, and the second crank \( O_B B \) is link 4, each with respective lengths \( a_2, a_3, a_4 \), and respective angles \( \theta_2, \theta_3, \) and \( \theta_4 \). The angle of each link is measured in a counterclockwise sense from the positive direction of the \( x \)-axis.

Figure 1.—Configuration of a four-bar linkage with coupler point \( C \).
The loop equations for the coordinates of B are written in the counterclockwise and clockwise sense as follows:

\[
\begin{align*}
  x_B &= x_{OB} + a_4 \cos \theta_4 \\
  &= x_{OA} + a_2 \cos \theta_2 + a_3 \cos \theta_3 \\
  y_B &= y_{OB} + a_4 \sin \theta_4 \\
  &= y_{OA} + a_2 \sin \theta_2 + a_3 \sin \theta_3
\end{align*}
\]

eq. 1

In this application, the input is through the knee block (the coupler), so the independent variable is taken to be \( \theta_3 \). Equations 1 and 2 are rewritten with one dependent variable, \( \theta_2 \), to the left of the equality and with the remaining quantities on the right.

\[
\begin{align*}
  a_2 \cos \theta_2 &= a_4 \cos \theta_4 + C_1, \\
  a_2 \sin \theta_2 &= a_4 \sin \theta_4 + C_2,
\end{align*}
\]

eq. 3

where,

\[
\begin{align*}
  C_1 &= x_{OB} - x_{OA} - a_3 \cos \theta_3, \\
  C_2 &= y_{OB} - y_{OA} - a_3 \sin \theta_3,
\end{align*}
\]

eq. 5

are both constants for any input angle \( \theta_3 \).

Equations 3 and 4 are squared and added to yield equation 7,

\[
\begin{align*}
  a_2^2 &= a_4^2 + C_1^2 + C_2^2 + 2C_1 a_4 \cos \theta_4 \\
  &+ 2 C_2 a_4 \sin \theta_4.
\end{align*}
\]

eq. 7

After rearranging and collecting terms, this results in the equation of motion:

\[
A \sin \theta_4 + B \cos \theta_4 = C,
\]

where:

\[
\begin{align*}
  A &= 2 C_2 a_4, \\
  B &= 2 C_1 a_4, \\
  C &= a_2^2 - a_4^2 - C_1^2 - C_2^2,
\end{align*}
\]

eq. 8

are constants for any input value of \( \theta_3 \), and \( C_1 \) and \( C_2 \) are defined in equations 5 and 6.
Equation 8 is of little direct use because it is an implicit transcendental function of $\theta_4$. This is made an explicit equation by substituting:

$$\sin \theta_4 = \frac{2 \tan \left( \frac{\theta_4}{2} \right)}{1 + \tan^2 \left( \frac{\theta_4}{2} \right)},$$  \hspace{1cm} \text{eq. 12}$$  

$$\cos \theta_4 = \frac{1 - \tan^2 \left( \frac{\theta_4}{2} \right)}{1 + \tan^2 \left( \frac{\theta_4}{2} \right)}.$$  

Equation 8 reduces to a quadratic in $\tan \left( \frac{\theta_4}{2} \right)$ with the solution:

$$\theta_4 = 2 \tan^{-1} \left( \frac{A \pm \sqrt{A^2 + B^2 - C^2}}{B + C} \right),$$  \hspace{1cm} \text{eq. 13}$$

There are two solutions for $\theta_4$, one for the positive sign and one for the negative sign of the square root term. Figure 2 portrays the two solutions as they occur in the linkage. For any one mechanism, the correct sign must be used or the remaining solutions will be meaningless.

At this time $\theta_3$ has been specified and $\theta_4$ has been computed. We can now find the coordinates of point B and point A.

$$x_B = x_{OB} + a_4 \cos \theta_4, \hspace{1cm} \text{eq. 14}$$  

$$y_B = y_{OB} + a_4 \sin \theta_4, \hspace{1cm} \text{eq. 15}$$  

$$x_A = x_B - a_3 \cos \theta_3, \hspace{1cm} \text{eq. 16}$$  

$$y_A = y_B - a_3 \sin \theta_3. \hspace{1cm} \text{eq. 17}$$

Knowing the coordinates of A and $O_A$, we can compute the angle $\theta_2$:

$$\theta_2 = \tan^{-1} \frac{y_A - y_{OA}}{x_A - x_{OA}}. \hspace{1cm} \text{eq. 18}$$

From the coordinates of the crank centers and the crank angles, the coordinates of the instant center, I, can be found from the geometry of Figure 1. After reducing the equations, the x- and y- coordinates of the instant center are:
\[ x_1 = x_{OA} + \frac{y_1 - y_{OA}}{\tan \theta_2} \quad \text{eq. 19} \]

\[ y_1 = \frac{y_{OB} + \left( x_{OA} - x_{OB} - \frac{y_{OA}}{\tan \theta_2} \right) \tan \theta_4}{1 - \frac{\tan \theta_4}{\tan \theta_2}}. \quad \text{eq. 20} \]

**Figure 2.** — Two solutions of a four-bar linkage for any coupler angle \( \theta_a \).
The coordinates of a coupler point will be required at a later time. Suppose this point is the point C in Figure 1. This point is a distance $a_C$ from A and an angle $\phi$ from the line AB. The angle and coordinate are written as follows:

$$\theta_C = \theta_3 + \phi,$$  \hspace{1cm} eq. 21

$$x_C = x_A + a_C\cos\theta_C,$$  \hspace{1cm} eq. 22

$$y_C = y_A + a_C\sin\theta_C.$$  \hspace{1cm} eq. 23

These equations define the motion of the mechanism and of any point, C, on the coupler.

**APPLICATION OF THE FOUR-BAR LINKAGE TO PROSTHESSES**

Let us now consider the applications of a four-bar linkage as a knee mechanism. The shank of the prosthesis is considered to be the frame linkage, and the socket and knee block constitute the coupler (Fig. 3a). The center of rotation of the coupler relative to the frame is the instant center I. As the coupler rotates to the left, as it would for knee flexion, it is required that the instant center moves from a position high and posterior to the TKA line downward to a position just below the anatomical knee axis so that at 90 deg. flexion the knee is cosmetically pleasing. This is described by the moving centrode rolling on the fixed centrode of the linkage. The centrodes are portrayed to a larger scale in Figure 3b. As the moving centrode rotates clockwise, the instant center (point of contact) moves down along the fixed centrode. The point $C_M$ on the moving centrode will move as shown forming a cusp at $C_F$. A general point such as D will move on a path as shown. The point $C_M$ is seen to have mostly horizontal motion, whereas point D has both horizontal and vertical motion. The two points depict a disadvantage and an advantage of the four-bar knee. The horizontal component of motion is undesirable because this relative motion of the knee block to the shank causes a gap to form between them as the knee flexes. The vertical component of displacement may be desirable if it causes the length of the prosthesis to shorten, thus making toe clearance easier during the swing-through phase of walking. It becomes obvious upon examination of the centrodes that the horizontal displacement cannot be eliminated if there is to be any vertical shift of the instant center, but it can be reduced if the instant center were to move down quickly as the knee flexes. This occurs when the two centrodes have nearly the same curvature when the knee is in full extension, and as the knee flexes, the curvature of the moving
Changes at an ever increasing rate. A balance must be maintained, for if the shift of instant center is too rapid there is a quick change in stability and the amputee cannot control knee flexion as well.

The relative horizontal displacement between the knee block and shank can be made less obtrusive by contouring the knee block so that a gap does not occur. This is done by considering the inversion of the four-bar and by drawing the coupler curve so that the upper lip of the shank cover traces out the knee block.

**Figure 3.**—a. Application of a four-bar linkage as a polycentric knee mechanism. b. Action of fixed and moving centrode.
The equations outlined in this paper have been computerized in such a manner that a four-bar linkage can be analyzed. The position of the instant center is computed for various coupler angles and the displacements of a coupler point such as C are computed. The motions of the linkage can now be compared to the desired motion by some criterion function and a series of linkage parameters systematically adjusted until the criterion function is either minimized or within desired limits.

The changing of the parameters must be observed and regional constraints applied such that the final mechanism conforms to them.

Any optimization method, such as Rosenbrock’s method of rotation of coordinates (1) and Powell’s method (4) or Fletcher and Reeves method (5), may be used for finding the best linkage.

Any number of criterion functions can be used depending upon the requirements of the problem. The criterion function used was:

\[
VF = e_1 U + e_2 x_{HD} + e_3 y_{HD}
\]

where
- \(VF\) is value of the criterion function
- \(U\) is maximum distance from the instant center at any input angle to the desired position of I at that angle
- \(x_{HD}\) is the maximum horizontal displacement
- \(y_{HD}\) is the maximum vertical displacement
- \(e_1, e_2, e_3\) are variable coefficients that can be changed from one problem to another.

The regional constraints were such that the linkage must always lie within the confines of the prosthesis and no link could be shorter than a predetermined length. A Grashof check (6) was always made and only crank-type mechanisms were allowed \((l + s < p + q)\). In this way a solution was always guaranteed as the parameters were changed.

Eight parameters were varied. These were: \(x_{0A}, y_{0A}, x_{0B}, y_{0B}, a_2, a_3, a_4,\) and the starting value of \(\theta_3\).

In using the program, the desired centrode is read into the computer as a set of coordinates of several points on the curve. The desired minimum value of the criterion function and the coefficients \(e_1, e_2, e_3\) are read in. A guess is made of a mechanism that might satisfy the conditions.

The mechanism is moved through the required range of coupler motion. The instant center and the coordinates of the coupler point C, corresponding to the location of the anatomical knee, are computed. The value of the criterion function is calculated and compared with the desired minimum. If the value is too large, the values of the eight
Parameters are systematically changed by the optimization subroutine; if not, the values of the parameters are printed along with the coordinates of both the desired and the actual centrodes. Finally, the knee block contour for some point Q on the shank is computed and printed. The optimum position of Q could also be found to give some desired knee block contour.

The final linkages found by this technique depend upon a great number of factors, the prime ones being the quality of the initial guess and the ultimate suitability of the desired centrode.

It is therefore required that a possible centrode be read into the computer and that the initial guess be a good one. The final optimization is usually carried through a series of improved initial guesses until a suitable solution is found. No attempt has been made to find an ultimate optimum for the linkages. The type of mechanism one can expect is all that has been attempted.

Example 1: Knee Linkage for Above-Knee Amputees

An attempt was first made to reduce the horizontal displacement, that is inherent in the Berkeley Knee described by Radcliffe (1), while specifying a desired centrode. Figure 4 depicts the desired centrode, the initial guess (shown as a dashed line), and the final linkage and its centrode. The motion of this linkage through five displacements of 20 deg. increments and the path of the knee center are shown in Figure 5, as well as the contouring of the knee block so that a gap is not formed at the top of the shank cover. A different criterion function would have given a different linkage with other optimums—for example, exact following of a centrode, or exact position of the knee at 90 deg. flexion. These can all be designed through the choice of a criterion function.

Example 2: Knee-Disarticulation Amputee

Amputation through the level of the knee results in an entirely different problem than the above-knee amputee. This amputation generally produces a good, "trouble-free" stump, as all major thigh muscles remain intact and the femoral condyles provide an excellent weight-bearing area. When well fitted with a conventional prosthesis, the amputee normally ambulates with a normal-appearing gait. The problem with a knee disarticulation is primarily cosmetic; the conventional side joints produce an exceptionally wide knee. Because the major hip flexor and extensor muscles function in a near normal manner, the effective knee center does not have to be high as in the above-knee amputation.

It was decided to use a crossed four-bar linkage which was located entirely below the knee block. The desired instant center moves from
FIGURE 4.—Desired centrode and initial guess (dashed) are optimized by program to give the final linkage and its centrode.
below the knee and traces a centrode behind and above the knee so that at 90 deg. the knee appears normal; to be competitive with the conventional knee, the horizontal displacement must be less objectionable than
the side bearings. The entire mechanism should fall within the confines of the shank to allow the full length of the stump. This approach has also been taken by a research team at the Orthopaedic Hospital, Copenhagen, Denmark (7).

Figure 6 shows one linkage that can be used for this situation complete with the path of knee center C. The knee center moves approximately 0.9 in. posteriorly and 0.5 in. downward. This horizontal motion is tolerable from the cosmetic point of view and the vertical motion shortens the limb for toe clearance. Another point in its favor is that slots are not required in the visible portion of the knee block for links to move in, as is the case in Figure 5.

If a rigid shank is used, the knee block contour shown is rather poor. This might be improved by optimizing the point on the shank or by using a flexible shank cover. Furthermore, the crank pin B is rather close to the knee center C. This indicates that another optimization should be carried out in which regional constraints are applied so that neither A nor B is too close to the knee center C.

**THE FINAL DESIGN**

Ultimately, the theoretical computer solution must be examined to see if it is suitable for the problem at hand. Such factors as limits of motion, transmission characteristics, acceptable lengths of links, and force and torque transmissions must be carefully analyzed.

Should the final mechanism not be suitable after a graphical check has been made, one of several alternative approaches can be taken:

1. The initial guess can be changed.
2. The shape of the fixed centrode can be changed.
3. The increment of angle θₙ can be changed.
4. The distribution of the design points on the centrode can be changed.
5. The increments used in changing the linkage parameters can be altered.
6. The criterion function can be altered to yield a better approach to the desired characteristics.

Once a satisfactory linkage configuration has been arrived at, the next phase entails the development of hardware for clinical assessment. This step involves detailed force analysis for proportioning the size of parts and bearings, knee block design, and final prototype fabrication. In addition to several other factors, the swing control system will also dictate, to a degree, the final mechanical configuration.
CONCLUSIONS

The theoretical approach to problems of this nature is only of benefit to the disabled person if it paves the way toward the development of improved appliances for his ultimate use. This paper outlines one
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Theoretical method which may be used as a design tool for the synthesis of four-bar linkages. The apparent advantages of the method outlined may be summarized as follows:

1. An optimized four-bar linkage can be synthesized which has a centrode closely approximating a specified curve.
2. While optimizing a specified centrode, the horizontal or vertical displacement, or both, can also be optimized.
3. In design of knee mechanisms, the contour of the knee block is precisely calculated so that the mate with the front portion of the shank is accurate throughout the functional range of knee flexion.
4. With minor changes in the program, coupler curve generators or function generators could be designed.

Finally, it should be noted that the method outlined here is a design tool. The two examples are used to demonstrate some applications of the design tool and should not be considered as a final configuration.

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